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A Manual of Geometrical Crystallography

TREATING SOLELY OF THOSE PORTIONS OF THE SUBJECT USEFUL IN THE IDENTIFICATION OF MINERALS

BY

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PREFACE

CRYSTALLOGRAPHY may be studied with two distinct purposes in view. The end usually sought is the ability to describe crystals with such detailed accuracy that minor variations between them and other crystals may be detected and shown graphically or embodied in mathematical expressions. This aspect of the subject belongs unquestionably in the realm of pure science. It involves the accurate measurement of angles with delicate instruments requiring careful manipulation, and the results secured are not sought with any idea that they may have practical value.

Quite different is the other purpose to which reference has been made, since it is the attainment of the ability to recognize crystal forms and especially systems almost instantly with the use of few if any instruments, and those who seek this knowledge do so wholly because they expect to use it as a tool for identifying minerals.

It is of the phase of the subject last mentioned that this book treats, and it is hoped that it will fill the needs of the growing group of educators who realize the great importance of teaching "sight recognition" of minerals to engineering students or others who study mineralogy merely for its cultural value. In order to conserve the students' time and

energy, everything not germane to the end sought — the acquisition of information useful in the "sight recognition" of minerals — has been omitted, and no hesitation has been felt in departing from current usage when it seemed desirable in order to secure simplicity and clarity.

The system followed is not an untried experiment, but was introduced many years ago by Dr. H. B. Patton in the Colorado School of Mines where it has been taught with marked success, and from which it has been carried by graduates to a number of other institutions. This system includes the study of numerous wooden, cardboard, or plaster models of crystals, together with oral quizzes involving the instantaneous identification of the forms represented on such models, and discussions of the theoretical aspects of the subject. After models belonging to a certain system or group of systems have been studied and a sufficient knowledge of them revealed in the quiz the student takes up the determination of natural crystals of the same degree of symmetry; and the study of crystal models and of corresponding natural crystals alternate throughout the course. If this plan is followed, it will be necessary for the student to familiarize himself with the matter presented in Chapter IX before attempting to work with natural crystals.

The author desires to acknowledge his great indebtedness to Patton's "Lecture Notes on Crystallography," and, to a lesser extent, to Bayley's "Elementary Crystallography" for ideas and even definitions and descriptions embodied in this book. While most of the illustrations are original, many are

copied without other acknowledgment than this from Bayley's "Elementary Crystallography," Dana's "System of Mineralogy," and Moses and Parsons' "Mineralogy, Crystallography, and Blowpipe Analysis."

G. MONTAGUE BUTLER

TUCSON, ARIZONA, *September* 15, 1917.

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A Manual of Geometrical Crystallography

CHAPTER I

INTRODUCTORY CONCEPTIONS

A Mineral Defined.

A mineral is a naturally occurring, homogeneous, inorganic substance.

Diamonds, white mica (muscovite), native gold, and flint are illustrations of minerals; while a silver coin, granite (consists essentially of two minerals, orthoclase and quartz, so is not homogeneous), coal (is organic material), and window glass are not minerals although they belong to the so-called "mineral world" as distinguished from the "animal and vegetable worlds."

Structure of Minerals.

Molecules: Minerals, as well as all other substances, are made up of extremely small particles called molecules which in any homogeneous substance are all alike in composition, size, and weight, but which are unlike in the particulars mentioned in different substances. They are believed to be separated from one another to some extent even in the hardest and densest materials. There is a tendency

for each molecule to be held in position with respect to adjacent molecules by certain forces of mutual attraction, against which is opposed a tendency for each molecule to move in a straight line. The relative strength of these two tendencies is believed to determine whether a substance is a gas, a liquid, or a solid.

Amorphous Structure Defined: A substance is said to possess an amorphous structure or to be amorphous when its constituent molecules are arranged according to no definite fashion or pattern. Presumably they lie at unequal distances with respect to each other, and lines joining their centers do not meet in fixed angles. If a box of oranges be dumped helter-skelter into a basket and each orange be considered analogous to an enormously magnified molecule, a good conception of the structure of an amorphous substance can be obtained.

Natural and artificial glasses are excellent illustrations of amorphous materials, but not a few minerals also possess this structure.

Crystalline Structure Defined: A substance is said to possess a crystalline structure or to be crystalline when the constituent molecules are arranged in some definite fashion or pattern. A box of oranges of equal size packed in even rows and layers is analogous to a crystalline substance in which each orange corresponds to a molecule, but it should not be understood that all crystalline materials have a structure resembling in detail the illustration just given.

Crystalline substances, while they may resemble amorphous ones very closely, at least superficially,

can usually be recognized by the presence of cleavage (see p. 142) or distinctive optical, electrical, thermal, or other physical properties which prove that in crystalline materials there are certain directions along which forces or agents act with quite different effects from those produced in other directions. Thus a sphere of glass (amorphous) when heated expands equally in all directions and remains perfectly spherical; while a sphere of emerald (crystalline) if similarly heated will be distorted and will become ellipsoidal due to the fact that the coefficient of expansion in one direction differs from that in all others.

The majority of minerals as well as many artificial substances have crystalline structures.

A most useful characteristic of a crystalline substance results from the fact that at the time of its formation it shows a more or less pronounced tendency to form a body bounded wholly or partially by plane surfaces or faces. Such a partial or complete polyhedron is called a crystal; and, if several such crystals develop in contact with or close proximity to one another, a group of crystals results.

While no simple definition distinguishing between single crystals and crystal groups can be offered, there should be little chance of a misconception arising through the use of the following definition.

A Crystal Defined.

A crystal is a crystalline substance bounded wholly or partially by natural plane surfaces called faces which have not been produced by external forces.

From what has been said it must be evident that a crystal always has a crystalline structure, but it is equally important to remember that crystalline substances do not by any means always occur in crystals. These are the exception rather than the rule, and develop only when conditions are favorable. When faces are lacking, other features (such as the presence of cleavage) or physical tests must be used to determine whether a substance is crystalline or amorphous.

Formation of Crystals: Crystals may form in two ways, namely, through deposition from solutions (including fusions which are now recognized as forms of solution) and from the sublimed (gaseous) condition. In either case, a solid molecule having formed, growth occurs through the addition of myriads of other molecules which surround the first according to some definite geometric plan. If the resulting crystal is in suspension in a gas, vapor, or liquid, it will be entirely bounded by crystal faces; otherwise, only those portions that are surrounded by the gas, vapor, or liquid will develop in the manner outlined.

Crystals are often formed in the manufacture of artificial substances, and these are subject to the same laws that apply to mineral crystals.

Crystallography Defined.

Crystallography is the science that deals with crystals.

Three branches of this science are recognized. These are geometrical crystallography, physical crystallography, and chemical crystallography. The scope of each is suggested by its name. The student

of determinative mineralogy is most concerned with the first of these branches, and this manual deals almost entirely with that phase of the science.

The study of crystals has great practical value to a mineralogist since it has been found that each crystalline mineral occurs in crystals whose shapes resemble each other very closely, and are, indeed, frequently almost identical no matter where found. Further, it is true that crystals of different minerals are usually quite dissimilar, and it is often possible for one familiar with crystallography to distinguish easily between two crystallized minerals which, except for the difference in their crystals, resemble each other very closely. Crystallographic terms are also employed in describing features used as criteria in determinative mineralogy.

FUNDAMENTAL DEFINITIONS

Some of the definitions that follow apply only to geometrically perfect crystals or crystal models. In the cases of incomplete and distorted crystals (discussed later) these conceptions will have to be modified as suggested in the concluding chapter.

A Symmetry Plane Defined.

A symmetry plane is any plane which divides an object in such a way that *any* line drawn perpendicular thereto, if extended in both directions, will strike the exterior of the object in similar points which are equidistant from the dividing plane.

Thus, in Fig. 1, AA' is a symmetry plane because a perpendicular drawn to it at any point, as at B,

strikes the exterior at C and at C' which are similar points equidistant from the plane. MM' is not a symmetry plane, however, since a perpendicular erected to it at N strikes the exterior at O and O'

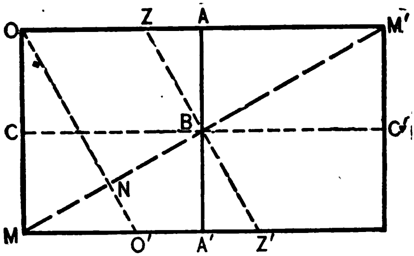


FIG. 1.

which are neither similar points (one is at a corner and the other lies on an edge) nor are they equidistant from the plane under consideration. It should be observed, however, that the perpendicular erected at B strikes the exterior at Z and Z' which *are* similar points and are equidistant from MM'. The definition states, however, that, in order that a given plane shall be a symmetry plane, the test given must be applicable to *any perpendicular one chooses to select*, and it has already been shown that it does not hold in the case of OO'.

The object used in the illustrations just given is a surface, but the same considerations apply to symmetry planes in solids.

Another definition of a symmetry plane especially useful in the case of solids is the following:

A symmetry plane is any plane so situated that, if it were a mirror, the reflection of the portion in

front of the mirror would seem to coincide exactly with the part behind.

From what has been said it is easy to see that, while symmetry planes divide objects into halves identical in shape and size, the mere fact that an object is so divided does not prove that the dividing plane is a symmetry plane. In Fig. 1 plane *MM'* divides the object into equal halves, but is not a symmetry plane.

As a corollary of the foregoing, it may be said that a symmetry plane is any plane that divides an object in such a way that every edge, corner, and face on one side of the plane is exactly balanced by identical edges, corners, and faces *directly opposite* on the other side of the plane.

All symmetry planes may be called either principal symmetry planes or secondary (sometimes called common) symmetry planes, as is explained later.

A Symmetry Axis Defined.

A symmetry axis is the line or direction perpendicular to a symmetry plane and passing through the center of the object.

A Principal Symmetry Plane Defined.

A principal symmetry plane is a symmetry plane perpendicular to which lie at least two *interchangeable* symmetry planes (either principal or secondary).

It should be remembered that there are three parts to this definition, and that any principal symmetry plane must conform to *all* of them. First, it must divide an object symmetrically — be a symmetry plane as already defined. Second, at

least two other symmetry planes existing in the object must be perpendicular to the plane under consideration (these need not be perpendicular to each other). Third, the two or more symmetry planes perpendicular to the one under consideration must be interchangeable. This third condition is the one most frequently misunderstood or overlooked by beginners. Attention should, then, be especially directed to the following paragraph.

Interchangeable Symmetry Planes and Axes Defined.

Two symmetry planes or axes are said to be interchangeable when one plane or axis may be placed in the position of the other plane or axis without apparently altering the appearance or position of the object.

A Principal Symmetry Axis Defined.

A principal symmetry axis is a symmetry axis perpendicular to a principal symmetry plane.

A Secondary (or Common) Symmetry Plane Defined.

A secondary symmetry plane is any symmetry plane that does not possess the characteristics of a principal symmetry plane as already defined.

A Secondary Symmetry Axis Defined.

A secondary symmetry axis is a symmetry axis perpendicular to a secondary symmetry plane.

An Interfacial Angle Defined.

An interfacial angle is an angle formed at the intersection of two faces or the planes of two faces. It must be measured perpendicular to the edge

formed by the intersection of the two faces; or, if the faces do not intersect in an edge, the measurement must be made perpendicular to the imaginary line located at the intersection of the planes of the two faces.

A Zone Defined.

A zone is a group of faces in the form of a belt or band which extends around a crystal in such a way that the edges formed by the mutual intersections of the faces are all parallel.

A Zonal Axis Defined.

A zonal axis is a line through the center of a crystal parallel to the faces of a zone.

Replaced Edges and Corners Defined.

A face is said to replace an edge when that face is substituted for, and lies parallel to, the edge, yet is *not equally inclined* to the two faces whose intersection would form that edge.

Similarly, a face may be said to replace a corner formed by the intersection of three or more faces when it is substituted for that corner, but is *not equally inclined* to at least one set of similar faces whose intersection would form that corner.

Truncated Edges and Corners Defined.

A face is said to truncate an edge when it is substituted for that edge in such a way as to be parallel to it and to make *equal angles* with the faces whose intersection would form that edge. Fig. 2 shows a crystal with truncated edges.

Similarly, a face may be said to truncate a corner when it is substituted for that corner and makes *equal angles* with all similar faces whose intersections would form that corner. Fig. 2 shows a crystal with truncated corners.

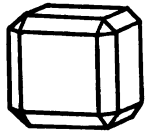


FIG. 2. — Hexahedron (cube) with edges truncated by the dodecahedron, and corners truncated by the octahedron.

Beveled Edges Defined.

Two faces are said to bevel an edge if they replace the edge in such a way that equal angles are formed between each replacing face and the adjacent faces whose intersection would form the edge. Fig. 3 shows a crystal with beveled edges.

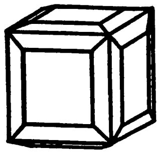


FIG. 3. — Hexahedron (cube) with edges bevelled by the tetrahexahedron.

A Crystal System Defined.

All those crystals which contain the same number and kind of symmetry planes (together with others produced by the suppression of certain faces in accordance with definite laws, which may be regarded as modifications of these) are said to belong to the same crystal system.

Number and Names of the Crystal Systems.

Any crystal may be placed in one of six crystal systems. Of these there are 32 subdivisions or classes, but few or no minerals are known to occur in some of these, so familiarity with all of them is unnecessary.

The six crystal systems are as follows:[1]

Number and kind of symmetry planes in crystals with the fully developed symmetry of the system.		Name of the system.
Principal.	Secondary.	
3	6	Isometric
1	6	Hexagonal
1	4	Tetragonal
0	3	Orthorhombic
0	1	Monoclinic
0	0	Triclinic

Any one crystalline mineral species always occurs in crystals characterized by the presence of a definite number and kind of symmetry planes, which is equivalent to saying that its crystals always belong to a certain one of the subdivisions of the above-mentioned crystal systems. Crystals containing the same kind and number of symmetry planes are said to show the same *degree of symmetry*.

[1] As it is absolutely necessary that the student should be able to recognize symmetry planes and to distinguish between principal and secondary symmetry planes, it is desirable that, before proceeding further, a number of crystal models should be separated, first into three groups each containing the same number of principal symmetry planes, and then into the six crystal systems, to do which the number of secondary symmetry planes must also be considered. Wooden models for this work are manufactured by Dr. F. Krantz, Bonn on Rhine, Germany, and sold at an average price of less than $0.50 each.

Designating the Position of Planes in Space.

The position of any plane may be defined by ascertaining its relation to three fixed lines or axes intersecting in a common point called the *origin*. This may be done by determining the distance and direction from the origin to the point at which the plane cuts each axis. As crystals are bounded by faces which are circumscribed portions of planes, the positions of such faces may be given by referring them to such a system of axes.

Crystal Axes Defined.

Crystal axes are fixed lines or directions to which crystal faces are referred for the purpose of ascertaining their mutual relationships.

General Rule for Choosing Crystal Axes.

Select the crystal axes so that they coincide where possible with symmetry axes, giving the preference to principal symmetry axes; but when an insufficient number of symmetry axes are present choose lines passing through the center of the crystal and parallel to prominent crystallographic directions, preferably edges. The crystal axes should intersect at as nearly right angles as possible in all systems but the hexagonal in which it is convenient to have the horizontal axes intersect at angles of 60° and 120°, instead of 90°.

The Terms Crystal and Symmetry Axes not Synonymous.

Care should be exercised not to confuse the terms crystal axes and symmetry axes. While it is true

that these sometimes coincide, they do not by any means always do so. Some symmetry axes are never used as crystal axes; and many crystal axes are not symmetry axes at all.

Designation and Use of Crystal Axes.

It is customary to call the crystal axis extending from front to back the *a* axis, the one from right to left the *b* axis, and the vertical one the *c* axis. Interchangeable crystal axes are represented by the same letter, however.

It will later be made plain that the faces on any crystal may be separated into groups each of which is characterized by the fact that its faces (or the planes of the faces) intersect all the crystal axes at distances from the origin which bear the same fixed ratios to each other. It has been observed that, in the case of any given mineral species, certain such groups are comparatively common while others are less common or rare. It is the usual presence of certain such groups of faces that causes the crystals of any given species to resemble each other very closely, and makes it possible to classify instantly many minerals occurring in crystals.

Ground-form or Unit-form Defined.

The ground-form or unit-form of the crystals of any mineral species is the most commonly occurring group of faces that intersect all the crystal axes at finite distances from the origin which distances bear the same fixed ratios to each other. In the isometric system the octahedron (see p. 19) is called the ground-form.

Unit Axial Lengths Defined.

The distances from the origin at which the faces (or faces extended) of the ground-form intersect the crystal axes are considered the unit axial lengths of those axes, provided that such a scale be used as will make the length of at least one of these axial lengths unity. This definition applies to all systems but the isometric. In that system the unit axial length is the shortest one of the three distances measured from the origin to the points where a face (or the plane of a face) intersects the crystal axes.

Practically, of course, any scale can be used in making the measurements mentioned in the last paragraph, since, for instance, if the distances measured in any scale on the *a*, *b*, and *c* axes are, respectively, 1.817, 1.112, and 1.253, and it is desired to have the *b* axial length unity, this may be brought about without affecting the ratio between the expressions by dividing each of the three expressions by the value for *b*. If this is done, the results will be 1.634, 1.000, and 1.125 which are the unit axial lengths of the crystals of the mineral selected as an illustration (sylvanite).

The letters *a*, *b*, and *c* are used not only to designate the crystal axes, but also to represent the unit axial lengths of these axes.

The distances from the origin at which faces (or faces extended) other than those belonging to the ground-form cut the axes are expressed in terms of the unit axial lengths, as $4a$, $2b$, and $1c$. If it is desired to have unity for the coefficient of *b*, this may be secured by dividing each expression by the

coefficient of b, which reduces the expressions to $2a$, $1b$, and $\frac{1}{2}c$.

A Parameter Defined.

A parameter is the distance from the origin to the point where a face (or a face extended) cuts a crystal axis, measured in terms of the unit length of that axis. Thus, in the illustration given in the preceding paragraph, 2, 1, and $\frac{1}{2}$ are the parameters of the face under consideration on the a, b, and c axis, respectively. It is customary to use m, n, and p as general expressions for parameters. A face parallel to an axis will intersect that axis at infinity, and will have infinity (∞) for its parameter on that axis.

The Law of Rationality of Parameters.

Parameters are always rational, fractional or whole, small or infinite numbers.

Crystal Form Defined.

A crystal form is a group of faces with identical parameters all of which are necessary to complete the symmetry of the system.

In explanation of this definition, it may be stated that in studying any system of crystals, if we assume the presence of a face or plane of given parameters, there must be present a definite number of other faces with identically the same parameters in order that the complete symmetry of the system may be shown. Such a group of faces is technically known as a crystal form. It will later be shown that there are *seven* distinctly different forms in each system and in each subdivision of a system.

Crystal Form and Shape Differentiated.

The student should be careful not to confuse the terms "form" and "shape" as applied to crystals. A crystal may have the general appearance of a cube, for instance, yet bear no faces with the parameters characteristic of the crystal form known as the cube. It may still, with propriety, be said to have a cubical shape, although the crystal form known as the cube is not represented upon it.

Symbol Defined.

A symbol in the Weiss system, of which a slight modification is used in this book, is the product of the parameters of a face and the corresponding unit axial lengths, arranged in the form of a ratio, as $na : b : mc$.

Since every face of any one form has the same parameters and unit axial lengths, it follows that the symbols of any face may be regarded as the symbol of the form of which that face is a part.

Several other systems of symbols are in more or less widespread use, and are presented in the more extended textbooks on crystallography. Lists of such symbols without further explanation are in this book appended to the description of each crystal system.

Law of Axes.

The opposite ends of crystal axes (as well as of symmetry axes) must be cut by the same number of similar crystal faces similarly arranged.

The importance of this law will be understood when the monoclinic and triclinic systems are studied.

Holohedral, Hemihedral, and Tetartohedral Forms Defined.

Holohedral Forms: When a form has the full symmetry of the system to which it belongs (see p. 11) it is said to be holohedral.

Hemihedral (half) Forms: A hemihedral form may be conceived to be developed by dividing a holohedral form by means of a certain set or sets of symmetry planes into a number of parts, then suppressing all faces *lying wholly within alternate parts* thus obtained, and extending all the remaining faces until they meet in edges or corners.

Tetartohedral (quarter) Forms: Tetartohedral forms may be conceived to be developed from holohedral ones by the simultaneous application of two different types of hemihedrism. These may be regarded as the half forms of half forms.

Hemimorphic Crystals Defined.

A hemimorphic crystal is one in which the law of axes is violated so far as one crystal axis is concerned; that is, the opposite ends of one crystal axis *are not* cut by the same number of similar faces similarly arranged.

Comparatively few minerals occur as hemimorphic crystals.

CHAPTER II

ISOMETRIC SYSTEM

HOLOHEDRAL DIVISION

Symmetry.

The holohedral division of the isometric system is characterized by the presence of three interchangeable principal symmetry planes and six interchangeable secondary symmetry planes. The former intersect at angles of 90°, and the latter at 60°, 90°, and 120° angles. The two classes of symmetry planes are so arranged that every 90° angle between principal symmetry planes is bisected by a secondary symmetry plane.

The Selection, Position, and Designation of the Crystal Axes.

In accordance with the general rule (see p. 12), the crystal axes in the isometric system are chosen so as to coincide with the principal symmetry axes. There are, then, in the isometric system three interchangeable crystal axes which are at right angles to each other. One is held vertically, and one so as to extend horizontally from right to left; the other must then extend horizontally from front to back. Since all of these axes are mutually interchangeable, each is called an *a* axis.

When a crystal is so held that the crystal axes extend in the proper direction as viewed by the observer it is said to be *oriented*.

Orienting Crystals.

Holohedral isometric crystals are oriented by holding a principal symmetry plane so that it extends vertically from front to back; then rotating the crystal around the principal symmetry axis perpendicular to this plane until another principal symmetry plane extends vertically from right to left, and a third such plane lies horizontally. The crystal axes will then extend in the proper directions.

An Octant Defined.

An octant in all systems but the hexagonal is one of the eight parts obtained by dividing a crystal by means of three planes each of which contains *two* crystal axes.

Holohedral Isometric Forms Tabulated. LEARN

The holohedral isometric forms, together with other data relating to each, are given in the following table:

		Symbol.	Name.	Number of faces.
Three axes cut alike		$a : a : a$	Octahedron (Fig. 4)	8
Two axes cut alike	Two axes cut at a distance < the other	$a : a : ma$	Trisoctahedron (Fig. 5)	24
		$a : a : \infty a$	Dodecahedron (Fig. 6)	12
	Two axes cut at a distance > the other	$a : ma : ma$	Trapezohedron (Fig. 7)	24
		$a : \infty a : \infty a$	Hexahedron (cube) (Fig. 8)	6
Three axes cut unlike		$a : ma : na$	Hexoctahedron (Fig. 9)	48
		$a : ma : \infty a$	Tetrahexahedron (Fig. 10)	24

Notes. — In the isometric system, m and n are never less than unity.
In the isometric system, it is customary to abbreviate the symbols by omitting the letter a and the ratio sign as $1mn$ for $a : ma : na$.
The symbol $a : a : ma$, for instance, is read a, a, ma without mention of the proportion signs.

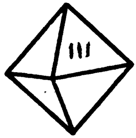


FIG. 4.—Octahedron.

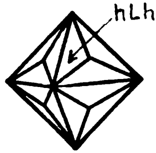


FIG. 5.—Trisoctahedron.

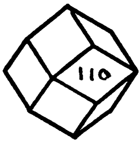


FIG. 6.—Dodecahedron.

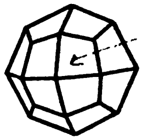


FIG. 7.—Trapezohedron.

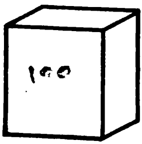


FIG. 8.—Hexahedron (cube).

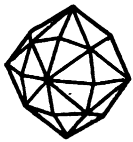


FIG. 9.—Hexoctahedron.

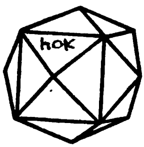


FIG. 10.—Tetrahexahedron.

Synonyms for the Names of the Holohedral Isometric Forms.

Octahedron—none.
Trisoctahedron—trigonal trisoctahedron.
Dodecahedron—rhombic dodecahedron.
Trapezohedron—icositetrahedron or tetragonal trisoctahedron.

Hexahedron — cube.
Hexoctahedron — none.
Tetrahexahedron — none.

Method of Determining Holohedral Isometric Forms by the Use of Symbols.

After properly orienting the crystal in the manner already described select any face in the upper right octant facing the observer, and ascertain the relative distances from the origin at which its plane cuts the three crystal axes. This may be done mentally or by laying a card upon the face and using a pencil to represent each axis in turn. If it appears that the three axes are cut equally, the symbol of the face (and of the form of which it is a part) is $a : a : a$. By referring to the table given on p. 19, which should be memorized as soon as possible, it is seen that the form is the octahedron. Similarly, if the plane of the face cuts one axis comparatively near the center of the crystal, and the other two at greater, but equal, distances, the symbol is $a : ma : ma$, and the form represented is the trapezohedron.

If more than one form is represented on the crystal (see p. 25), each may be determined in the same way. By using this method, it may be found that a crystal like Fig. 2 shows the hexahedron, dodecahedron, and octahedron; and, no matter how complex a crystal may be, the forms represented upon it may thus be readily ascertained.

After the name of a form has been determined by the method suggested the result may be checked by noting whether the form in question has the requisite

number of faces. To determine the number of faces, it is only necessary to count those lying in one octant and to multiply this sum by eight. If it is found, for instance, that three half ($1\frac{1}{2}$) faces lie within an octant, the form has eight times one-and-a-half, or twelve faces, and is a dodecahedron.

Suggestions for Attaining Facility in the Recognition of Forms.

While it is permissible and, in fact, almost necessary at first to use the symbols for the purpose of determining crystal forms, this method is too slow to be wholly satisfactory, and should quickly be displaced by the one outlined below, which has for its aim the instantaneous determination of forms through familiarity with the position or slope of one or more of their faces.

In order to use this method, orient the crystal, and study the face or faces of different shape or size in or near the upper right octant facing the observer. Then determine which of the following descriptions (which should be learned at once) apply to the face or faces seen.

Cube: A horizontal face on top of the crystal. The faces of the cube are parallel to the principal symmetry planes.

Dodecahedron: A face parallel to the right and left axis and sloping down toward the observer at a steep angle — 45° from the horizontal. The dodecahedron has three faces lying in the octant with an edge running from above the center of the octant up toward the vertical axis, but these faces do not lie *wholly within* the octant. (Compare with the tris-

octahedron.) The faces of the dodecahedron are parallel to the secondary symmetry planes.

Tetrahexahedron: A face parallel to the right and left axis and sloping down toward the observer at a relatively gentle angle — less than 45° from the horizontal. The tetrahexahedron has six faces lying in the octant, but they do not lie *wholly within* the octant. (Compare with the hexoctahedron.)

Octahedron: A single face in the center of the octant, sloping steeply down from the vertical axis — at an angle of $54\frac{3}{4}°$ with the horizontal.

Trapezohedron: Three faces lying *wholly within* the octant and so arranged that a *face* slopes above the center of the octant up toward the vertical axis at an angle less steep than that shown by the octahedron. It is often useful to remember, further, that two faces forming part of the same trapezohedron may intersect *below the center* of the octant in an edge that points directly toward the vertical axis.

The trapezohedron is most apt to be confused with the trisoctahedron, described below, and the two descriptions should be carefully compared.

Trisoctahedron: Three faces lying *wholly within* the octant and so arranged that in the unmodified form an *edge* slopes *above* the center of the octant up toward the vertical axis. Even when so modified that the edge is lacking, it is easy to see that two faces extended would intersect in such an edge.

Hexoctahedron: Six faces lying *wholly within* the octant. As with the trisoctahedron and dodecahedron, there is, on the unmodified form, an edge running above the center of the octant up toward the vertical axis.

Fixed and Variable Forms Defined.

A Fixed Form: A fixed form is one that has no variable parameter (m, n, or p) in its general symbol. The octahedron, dodecahedron, and cube are fixed holohedral isometric forms.

The fixed forms never vary in the slightest degree in appearance, and their interfacial angles are fixed quantities.

A Variable Form: A variable form is one that has one or more variable parameters (m, n, or p) in its general symbol. That is, the symbol contains one or more parameters to which various values, such as 2, $2\frac{1}{2}$, 3, etc., may be assigned without affecting the naming of the form. The trisoctahedron, trapezohedron, hexoctahedron, and tetrahexahedron are variable holohedral isometric forms.

Two or more variable forms of the same name may differ considerably in shape if the values of the variable parameters in their symbols are materially different. For instance, Fig. 11 shows two trapezohedrons that do not resemble each other very closely. This is because the symbol of the one to the left is $a : 2a : 2a$, while that of the one to the right is $a : 3a : 3a$.

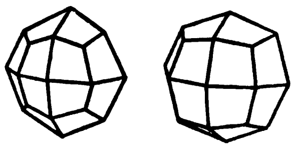


FIG. 11.— Trapezohedrons with symbols (on left) $a : 2a : 2a$ and (on right) $a : 3a : 3a$.

Interfacial Angles of the Fixed Forms.

Octahedron, 109° $28\frac{1}{4}'$ (usually given as $109\frac{1}{2}$°).
Dodecahedron, 60°, 90°, and 120°.
Cube, 90°.

The interfacial angles of the fixed forms are often called the *fixed angles* of a system.

Combination of Forms.

It will be found that, while some crystals are bounded by a single crystal form, the majority exhibit more than one such form. When this is the case the shapes of the faces shown in Figs. 4 to 10, inclusive, may be decidedly changed, but their symbols will remain unaltered.

Determination of the Number of Forms on a Crystal.

On a perfectly developed crystal there are as many forms as there are differently shaped and dimensioned faces. This is equivalent to saying that, whether unmodified or modified by the presence of other forms, all the faces of a given crystal form on a crystal are of identically the same shape and size.

Repetition of Forms on a Crystal.

Each of the fixed forms can occur but once on a crystal.

Each of the variable forms may occur an indefinite number of times on a crystal. Theoretically, one might say an infinite rather than an indefinite number of times, but, practically, the number of times a variable form is repeated on a crystal is usually small.

In order that a given variable form may occur more than once on a crystal, it is, of course, necessary that the symbols of the various forms of the same name differ as regards the values of the variable

parameter or parameters. Thus, the trapezohedron $a : 2a : 2a$ can occur but once on a crystal, but it may be combined with the $a : 1\frac{1}{2}\,a : 1\frac{1}{2}\,a$ trapezohedron, the $a : 3a : 3a$ trapezohedron, etc.

Holohedral isometric models showing repeated forms are difficult to obtain.

Learn

Modification of Fixed Forms.

It will be found useful to memorize the names of the forms which truncate and bevel the edges, and truncate the corners, of each of the fixed forms, as set forth in the following table:

Form modified.	Form truncating edges.	Form beveling edges.	Form truncating corners.
Octahedron	dodecahedron	trisoctahedron	cube
Dodecahedron	trapezohedron	hexoctahedron	cube and octahedron
Cube	dodecahedron	tetrahexahedron	octahedron

The Triangle of Forms.

It will be found advantageous to arrange the crystal forms around a triangle as shown in Fig. 12. The fixed forms are at the corners of this triangle, while the variable ones completely fill its sides and center. Theoretically there is an infinite number of trisoctahedrons along the left-hand side, each of which differs from the others as regards the magnitude of the variable parameter. Similarly the other sides of the triangle are filled with infinite numbers of trapezohedrons and tetrahexahedrons, while the interior of the figure should be conceived as completely filled with an infinite quantity of hexoctahedrons.

Utilization of the Triangle of Forms.

The triangle may be used to identify small, obscure forms replacing or truncating the edges between larger and more easily recognized ones, since a form lying in a straight line between two other forms on

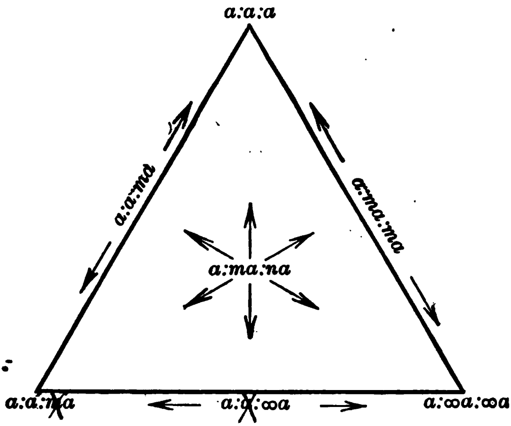


FIG. 12. — "Triangle of Forms."

the triangle will replace or truncate the edge between those same forms on a crystal. For instance, suppose a small face replaces the edge between a dodecahedron and a cube. Reference to the triangle shows that the only form lying in a straight line between the dodecahedron and the cube is a tetrahexahedron. The tetrahexahedron must then be the form whose name is sought. From what was said in the preceding paragraph, it must be evident that an indefinite number (theoretically infinite) of tetrahexahedrons may replace the edge between a cube and dodecahedron. Similarly, it may be

ascertained that nothing but hexoctahedrons can replace the edge between a trapezohedron and a dodecahedron, or between an octahedron and a tetrahexahedron; while the only form that can replace the edge between two trisoctahedrons is another trisoctahedron.

Limiting Forms Defined.

Limiting forms are those forms which a variable form approaches in appearance ("shape") as the variable parameter (or parameters) in its symbol approaches either unity or infinity. Thus, a trapezohedron ($a : ma : ma$) approaches an octahedron ($a : a : a$) in symbol and in shape as m approaches unity, and a cube ($a : \infty a : \infty a$) as m approaches ∞. The octahedron and cube are, then, said to be limiting forms of the trapezohedron.

Where a variable form is situated on one side of the triangle of forms, the two forms at the extremity of that side are its limiting forms. The hexoctahedron, in the interior of the triangle, has all the other six forms as its limiting forms.

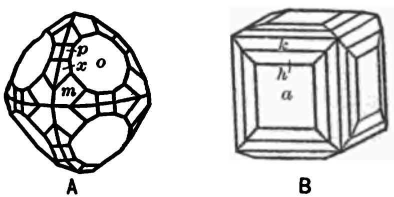


FIG. 13. — Holohedral isometric crystals.

A: Octahedron (o), trapezohedron (m), trisoctahedron (p), and hexoctahedron (x).

B: Cube (a), and two tetrahexahedrons (h and k).

TETRAHEDRAL (INCLINED) HEMIHEDRAL DIVISION

Development or Derivation of the Forms.

Tetrahedral hemihedral isometric forms may be conceived to be developed by dividing each holohedral form by means of the three principal symmetry planes into eight parts (octants), then suppressing all faces lying wholly within alternate parts thus obtained, and extending all the remaining faces until they meet in edges or corners.

Symmetry.

Tetrahedral hemihedral forms possess only the six secondary symmetry planes characteristic of the isometric system, since the method of development outlined necessarily destroys the principal symmetry planes.

In general, it may be said that *the planes used for dividing holohedral forms in the development of hemihedral or tetartohedral forms are always destroyed.*

Selection, Position, and Designation of the Crystal Axes.

The three directions used as crystal axes in the holohedral division are still utilized for the same purpose in the tetrahedral hemihedral division. In other words, three interchangeable directions at right angles to each other are used, one of which is held vertically, another extending horizontally from right to left, and the third extending horizontally from front to back. These are, however, no longer

symmetry axes, as they were in the holohedral division. Each is called an *a* axis, as in the holohedral division.

Learn

Orienting Crystals.

Tetrahedral hemihedral crystals are oriented by holding a symmetry plane (secondary) so that it extends vertically from front to back, then rotating the crystal around the symmetry axis perpendicular to this plane until a second symmetry plane extends vertically from right to left, and, finally, rotating the crystal around a vertical axis 45° either to the right or left. When this has been done the symmetry planes will occupy their proper positions, and the crystal axes will extend in the proper directions.

Tetrahedral Hemihedral Isometric Forms Tabulated.

Name.	Symbol.	Number of faces.	Form from which derived.
±Tetrahedron (Fig. 14)........	$\pm\frac{a:a:a}{2}$	4	octahedron
± Trigonal tristetrahedron (Fig. 15)	$\pm\frac{a:ma:ma}{2}$	12	trapezohedron
± Tetragonal tristetrahedron (Fig. 16)	$\pm\frac{a:a:ma}{2}$	12	trisoctahedron
± Hextetrahedron (Fig. 17)....	$\pm\frac{a:ma:na}{2}$	24	hexoctahedron
Hexahedron (cube) (Fig. 8).....	$\frac{a:\infty a:\infty a}{2}$	6	hexahedron (cube)
Dodecahedron (Fig. 6).........	$\frac{a:a:\infty a}{2}$	12	dodecahedron
Tetrahexahedron (Fig. 10)......	$\frac{a:ma:\infty a}{2}$	24	tetrahexahedron

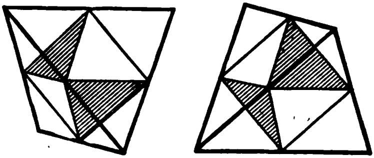


FIG. 14. — Positive (on left) and negative (on right) tetrahedrons containing the forms from which they are derived. The suppressed faces are shaded.

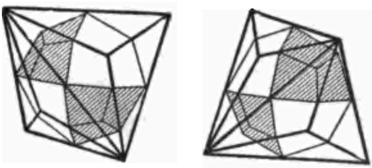


FIG. 15. — Positive (on left) and negative (on right) trigonal tristetrahedrons containing the forms from which they are derived. The suppressed faces are shaded.

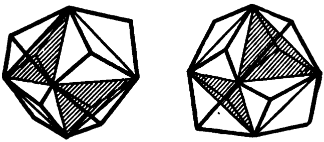


FIG. 16. — Positive (on left) and negative (on right) tetragonal tristetrahedrons containing the forms from which they are derived. The suppressed faces are shaded.

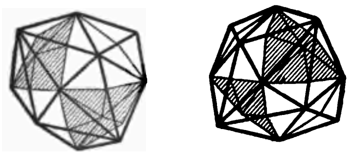


FIG. 17. — Positive (on left) and negative (on right) hextetrahedrons containing the forms from which they are derived. The suppressed faces are shaded.

Synonyms for the Names of the Tetrahedral Hemihedral Isometric Forms.

Tetrahedron — none.

Trigonal tristetrahedron — tristetrahedron or hemitetragonal trisoctahedron.

Tetragonal tristetrahedron — deltoid dodecahedron or hemitrigonal trisoctahedron.

Hextetrahedron — hemihexoctahedron.

Symbols of Hemihedral Forms.

The symbol of a hemihedral form is the same as that of the holohedral form from which it is derived excepting that it is written as a fraction with the figure 2 as the denominator. This does not mean that, in the case of hemihedral forms, the axes are intersected at half the holohedral axial lengths, but it is merely a conventional method of indicating that the symbol is that of a half (hemihedral) form. The symbol $\frac{a : a : a}{2}$ is read *a*, *a*, *a* over 2.

Positive (+) and Negative (−) Forms Distinguished.

All those forms produced by the suppression of faces lying within *the same set* of alternating octants are said to be of the same sign (+ or −). It is customary to consider those forms with faces largely or entirely included within the upper right octant facing the observer as +, while those with faces in the upper left octant are −. In reality, a + tetrahedron differs in no way from a − tetrahedron excepting in position; and a tetrahedron may be held in either the + or − position at will. It is customary to hold a crystal in such a way as to bring the larger and more prominent faces principally or

entirely into the upper right octant facing the observer, which will make these forms + ones.

If the sign of a form is not specifically stated to be −, it is always assumed that the form is +.

The forms on a crystal may be all of the same sign, or + and − forms may be combined.

If a + and a − form of the same name and with identical parameters are equally developed, the combination will have the exact external shape of a holohedral form. Thus, a + and a − tetrahedron equally developed will yield an octahedron. On natural crystals equally developed + and − forms often differ in that the faces of one may be brilliant, and the other dull; or one may be striated, and the other unstriated (see p. 138); or one may striate another form (see p. 139), and the other fail to do so.

Method of Determining Tetrahedral Hemihedral Isometric Forms by the Use of Symbols.

After properly orienting the crystal in the manner already described the cube, dodecahedron, and tetrahexahedron may be identified easily by applying the rules already given for the determination of holohedral forms of the same name. The other four forms may be recognized by determining the symbol of any face in the manner described in the discussion of holohedral forms, dividing this symbol by 2, and then ascertaining from the table the name of the form possessing this symbol.

Suggestions for Attaining Facility in the Recognition of the Forms.

Orient the crystal and determine which of the following descriptions (which should be learned at

once) apply to the face or faces of different shape or size seen. Call the forms + or − according to the rules already set forth. It is assumed that the student is already familiar with the rules for recognizing those forms identical in shape with the holohedral ones (see p. 22).

Tetrahedron: A single face in the center of the octant, sloping steeply down from the vertical axis — at an angle of $54\frac{3}{4}°$ with the horizontal.

Trigonal Tristetrahedron: Three faces in an octant (although not necessarily wholly included within the same), so arranged that a *face* slopes above the center of the octant up towards the vertical axis at an angle less steep than is shown by the tetrahedron.

Tetragonal Tristetrahedron: Three faces lying in an octant (although not necessarily wholly included therein), so arranged that in the unmodified form an edge slopes above the center of the octant up toward the vertical axis. Even when so modified that the edge is lacking, it is easy to see that two faces extended would intersect in such an edge. No faces of this form are parallel to any crystal axis *or to each other*. This will serve to distinguish this form from the one with which it is most easily confused, namely, the dodecahedron, the faces of which *are* arranged in *parallel pairs*.

Hextetrahedron: Six faces lying in an octant (although not necessarily wholly included therein), with no faces parallel to a crystal axis or to each other. The latter portion of this statement will serve to distinguish the form from the tetrahexahedron with which it may be most easily confused.

Distinction Between Hemihedral and Holohedral Forms of Exactly the Same Shapes.

The three forms last named in the table of tetrahedral hemihedral forms may differ in no way whatever from the corresponding holohedral forms so far as external shapes are concerned; and a model of a cube, for instance, may with equal propriety be considered either a holohedral or a hemihedral cube. On natural crystals, however, hemihedral cubes, dodecahedrons, and tetrahexahedrons differ in molecular structure and resulting physical properties from the holohedral forms of the same name. Holohedral and hemihedral forms of the same name may often be readily distinguished if striated by other forms (see p. 139).

Reason Why Some Developed or Derived Forms do not Differ in Shape from the Forms from which They Were Derived.

In general, it is true that a derived form is identical in shape with the form from which it was developed when no faces of the latter lie wholly within the parts obtained by dividing the form in the manner specified in the rule for developing the derived form.

In the tetrahedral division of the isometric system the three forms which fail to develop into others differing from the holohedral forms in shape are all characterized by the fact that they have infinity in their symbol. This indicates that each face is parallel to one or two of the axes, and must, therefore, lie in two adjacent octants. Since no face lies wholly within an octant, none can be suppressed in accordance with the rule given.

Law Governing Combination of Forms.

All the forms on any one crystal must possess the same degree of symmetry as regards their molecular structure.

The law just given is equivalent to the statement that it is impossible to have holohedral and hemihedral, or holohedral and tetartohedral forms on the same crystal; and that it is equally impossible for forms belonging to different hemihedral or tetartohedral divisions to occur together.

It has already been explained that some forms really hemihedral so far as their internal structure is concerned may be identical with holohedral forms in external appearance. These may, of course, be combined with the other hemihedral forms peculiar to the same division of the system. The same statement applies to hemihedral forms which are identical in appearance with hemihedral forms belonging to other divisions; and to tetartohedral forms identical in shape with those in other tetartohedral divisions or with hemihedral or holohedral forms.

Modification of Fixed Forms.

Form modified.	Form truncating edges.	Form beveling edges.	Form truncating corners.
+Tetrahedron...	cube	+trigonal tristetrahedron	−tetrahedron
−Tetrahedron...	cube	−trigonal tristetrahedron	+tetrahedron
Cube	dodecahedron	tetrahexahedron	±tetrahedron
Dodecahedron...	±trigonal tristetrahedron	±hextetrahedron	cube and ±tetrahedron

Miscellaneous.

All that was said in the discussion of the holohedral division concerning fixed and variable forms, interfacial angles of the fixed forms, combination of forms, determination of the number of forms, repetition of forms on a crystal, the triangle of forms, and limiting forms applies with equal truth to all hemihedral and tetartohedral divisions, excepting that the names of the derived forms must be substituted in these statements for those from which they were derived.

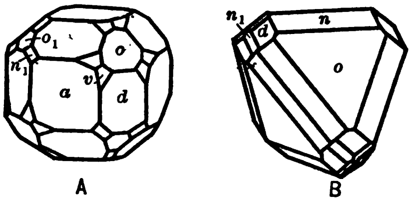


FIG. 18. — Tetrahedral hemihedral isometric crystals.

A: Cube (a), dodecahedron (d), + and − tetrahedron (o and o_1), + hextetrahedron (v), and − trigonal tristetrahedron (n_1).

B: + tetrahedron (o), + and − trigonal tristetrahedron (n and n_1), and dodecahedron (d).

PENTAGONAL (PARALLEL) HEMIHEDRAL DIVISION

Development or Derivation of the Forms.

Pentagonal hemihedral isometric forms may be conceived to be developed by dividing each of the holohedral forms by means of the six secondary symmetry planes into twenty-four parts, then suppressing all faces lying wholly within alternate parts thus

obtained, and extending the remaining faces until they meet in edges or corners.

Symmetry.

Pentagonal hemihedral forms possess only the three principal symmetry planes characteristic of the isometric system.

Selection, Position, and Designation of the Crystal Axes.

The three directions used as crystal axes in the holohedral and tetrahedral hemihedral divisions are still utilized for the same purpose in the pentagonal hemihedral division. In other words, three interchangeable directions at right angles to each other are used, one of which is held vertically, another extending horizontally from right to left, and the other horizontally from front to back. These coincide in position with the principal symmetry axes, and each is called an *a* axis, as in the holohedral division.

Orienting Crystals.

Pentagonal hemihedral crystals are oriented in exactly the same way as holohedral ones (see p. 19).

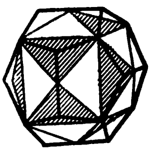


FIG. 19. — Pentagonal dodecahedron containing the form from which it is derived. The suppressed faces are shaded.

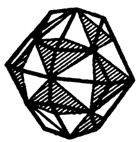


FIG. 20. — Diploid containing the form from which it is derived. The suppressed faces are shaded.

Pentagonal Hemihedral Isometric Forms Tabulated.

Name.	Symbol.	Number of faces.	Form from which derived.
Pentagonal dodecahedron (Fig. 19)	$\frac{a : ma : \infty a}{2}$	12	tetrahexahedron
Diploid (Fig. 20)	$\frac{a : ma : na}{2}$	24	hexoctahedron
Octahedron (Fig. 4)	$\frac{a : a : a}{2}$	8	octahedron
Dodecahedron (Fig. 6)	$\frac{a : a : \infty a}{2}$	12	dodecahedron
Hexahedron (cube) (Fig. 8)	$\frac{a : \infty a : \infty a}{2}$	6	hexahedron (cube)
Trapezohedron (Fig. 7)	$\frac{a : ma : ma}{2}$	24	trapezohedron
Trisoctahedron (Fig. 5)	$\frac{a : a : ma}{2}$	24	trisoctahedron

Synonyms for the Names of the Pentagonal Hemihedral Isometric Forms.

Pentagonal dodecahedron — pyritohedron.
Diploid — didodecahedron.

Method of Determining Pentagonal Hemihedral Isometric Forms by the Use of Symbols.

After properly orienting the crystal in the manner already described the octahedron, dodecahedron, cube, trapezohedron, and trisoctahedron may be identified easily by applying the rules already given for the determination of holohedral forms of the same name. The other two forms may be recognized by determining the symbols of any face in the manner described in the discussion of the holohedral forms, dividing this symbol by 2, and then ascertaining from the table the name of the form possessing this symbol.

Suggestions for Attaining Facility in the Determination of the Forms.

Orient the crystal and determine which of the following descriptions (which should be learned at once) apply to the face or faces of different shape or size seen. It is unnecessary to distinguish between positive and negative forms in this division. It is assumed that the student is already familiar with the rules for recognizing those forms identical in shape with the holohedral ones (see p. 22).

Pentagonal dodecahedron: A face parallel to the right and left axis, and sloping down toward the observer at a relatively gentle angle — less than 45° from the horizontal. The angle which this face makes with a horizontal plane is all that distinguishes it from the face of a dodecahedron with which it may most easily be confused.

If after orienting a crystal a prominent face is found sloping *steeply* down toward the observer — over 45° from the horizontal, rotate the crystal 90° to right or left around the vertical axis before applying the rule just given.

Diploid: Three faces lying wholly within the octant, so arranged that none of the edges formed by the intersection of the faces in the unmodified form point directly toward the vertical axis. This applies either to edges below or above the center of the octant. It is useful to remember, further, that edges formed by the intersection of a diploid face with an octahedron face, either below or above the center of the octant, are *never horizontal;* while a trapezohedron face does intersect an octahedron *above* the center on the octant in a *horizontal edge;*

and a trisoctahedron face intersects an octahedron face *below* the center of the octant in a horizontal edge. It should be noted further that trapezohedron faces intersect cube faces in edges which make right-angles with each other, while similar edges formed by the intersection of diploid and cube faces are never at right-angles. These statements should serve to distinguish the diploid from either the trapezohedron or trisoctahedron with which it is most easily confused.

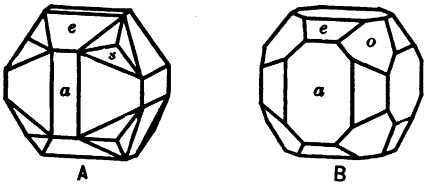


FIG. 21. — Pentagonal hemihedral isometric crystals.
A: Cube (*a*), pentagonal dodecahedron (*e*), and diploid (*s*).
B: Cube (*a*), octahedron (*o*), and pentagonal dodecahedron (*e*).

Application to the Law Governing Combination of Forms.

The law already stated (see p. 36) governing the combination of forms should not be forgotten when naming the forms on a pentagonal hemihedral crystal. One of the commonest mistakes made in determining crystal forms is to mention two or more forms which cannot possibly occur together on a crystal, as, for instance, a tetrahexahedron and a pentagonal dodecahedron. Any form in the pentagonal hemihedral division may be combined with

any other form in the same division, but no form in the pentagonal hemihedral division may be combined with any form in the tetrahedral hemihedral or holohedral divisions unless that form occurs also in those divisions. The student is very apt to make the mistake mentioned unless he can reproduce easily from memory the following table:

Holohedral forms.	Corresponding tetrahedral hemihedral forms.	Corresponding pentagonal hemihedral forms.
Octahedron......	*tetrahedron*	octahedron
Trapezohedron...	*trigonal tristetrahedron*	trapezohedron
Trisoctahedron...	*tetragonal tristetrahedron*	trisoctahedron
Hexoctahedron...	*hextetrahedron*	*diploid*
Tetrahexahedron.	tetrahexahedron	*pentagonal dodecahedron*
Dodecahedron ...	dodecahedron	dodecahedron
Cube............	cube	cube

It will be noted from the above table that the cube and dodecahedron occur in all three of the divisions already discussed, and may, therefore, be combined with any other forms in these divisions.

Further, it will be seen that the octahedron, trapezohedron, trisoctahedron, and tetrahexahedron occur unchanged in name or shape in two of the divisions; while the hexoctahedron occurs only as a holohedral form.

GYROIDAL HEMIHEDRAL DIVISION

Gyroidal hemihedral isometric forms may be conceived to be developed by dividing each holohedral form by both the three principal and the six secondary symmetry planes into forty-eight parts, then suppressing all faces lying wholly within

alternate parts thus obtained, and extending all the remaining faces until they meet in edges or corners.

As the hexoctahedron is the only isometric form with forty-eight faces, it is evident that a hexoctahedron face is the only one that can lie wholly within one of the parts obtained by dividing an isometric crystal in the manner just specified. The hexoctahedron is, then, the only isometric form from which a gyroidal hemihedral form differing from the holohedral one in shape and name can be derived. This new form is called the pentagonal icositetrahedron (Fig. 22). This form may be either right- or left-handed, but gyroidal hemihedral forms are so rare and unimportant that further discussion of them seems unnecessary.

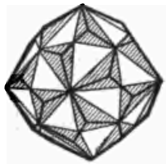


FIG. 22. — Pentagonal icositetrahedron containing the form from which it is derived. The suppressed faces are shaded.

PENTAGONAL TETARTOHEDRAL DIVISION

Pentagonal tetartohedral isometric forms may be conceived to be developed by the simultaneous application of any two hemihedrisms, according to the principles outlined in the discussion of the trapezohedral tetartohedral hexagonal division (see p. 72).

The only holohedral form which yields a tetartohedral form of different shape and name is the hexoctahedron, and the form derived from it is called a tetartohedral pentagonal dodecahedron, tetarto hexoctahedron, or tetartoid (Fig. 23). Both + and −, right- and left-handed tetartoids are distinguishable, but tetartohedral isometric crystals are so rare that further discussion of them seems unnecessary.

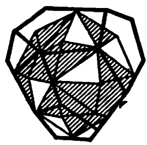


FIG. 23. — Tetartoid containing the form from which it is derived. The suppressed faces are shaded.

Table of Isometric Symbols Used by Various Authorities.

	Weiss.	Naumann.	Dana.	Miller.
Octahedron	$a : a : a$	O	1	(111)
Trisoctahedron	$a : a : ma$	mO	m	(hhl)
Dodecahedron	$a : a : \infty a$	∞O	i	(110)
Trapezohedron	$a : ma : ma$	mOm	m–m	(hll)
Hexahedron (cube)	$a : \infty a : \infty a$	∞O∞	i–i	(100)
Hexoctahedron	$a : ma : na$	mOn	m–n	(hkl)
Tetrahexahedron	$a : ma : \infty a$	∞On	i–n	$(hk0)$

Weiss, Naumann, and Dana divide the holohedral symbols by 2 and by 4 when referring to hemihedral and tetartohedral forms, respectively. Miller prefixes various Greek letters when forming the symbols of hemihedral and tetartohedral forms.

CHAPTER III

HEXAGONAL SYSTEM

HOLOHEDRAL DIVISION

Symmetry.

The holohedral division of the hexagonal system is characterized by the presence of one principal and six secondary symmetry planes which lie at right angles to the principal symmetry plane. The secondary symmetry planes are arranged in two groups each of which contains three planes. The planes of each group intersect each other at angles of 90° and 120°, and are interchangeable; while the planes of one group are non-interchangeable with those of the other group which they intersect at angles of 30°, 90°, or 150°.

Selection, Position, and Designation of the Crystal Axes.

The principal symmetry axis is chosen as one of the crystal axes, is held vertically, and is called the *c* axis. *Three* other crystal axes are so selected as to coincide with either group of interchangeable secondary symmetry axes. One is held horizontally from right to left. The other two will, then, be horizontal, but will make angles of 60° with each other as well as with the right and left axis (see Fig. 24). Since all three horizontal axes are interchangeable, they are all called *a* axes. None of the

horizontal axes is interchangeable with the vertical axis.

This is the only system in which more than three crystal axes are used. While it would be possible to determine the holohedral forms by the use of three

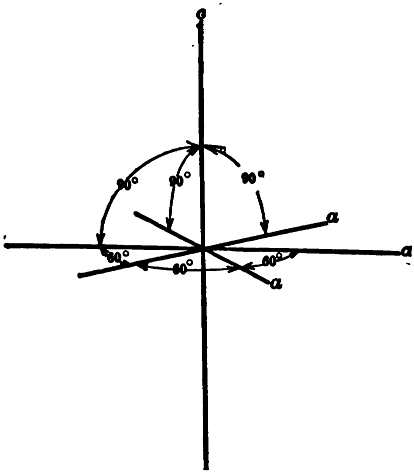


FIG. 24. — Crystal axes of the hexagonal system.

non-interchangeable axes intersecting at right angles, as in the orthorhombic system (see p. 103), this would make it necessary to attach different names to faces identically of the same shape and size, and would in no way suggest the six- or three-fold arrangement of faces which distinguishes this system. It would further necessitate the devising of new rules for developing hemihedral and tetartohedral forms; and would lead to so many difficulties that it is far simpler to use the three interchangeable horizontal axes than two non-interchangeable ones.

Orienting Crystals.

Holohedral hexagonal forms are oriented by holding the principal symmetry plane horizontally, and either set of interchangeable secondary symmetry planes in such a way that one of the planes will extend vertically from right to left. The crystal axes will then extend in the proper directions.

The Law of Rationality or Irrationality of Ratios Between Unit Axial Lengths.

The ratio between two unit axial lengths on non-interchangeable axes is always an irrational quantity; while the ratio between the unit axial lengths on interchangeable axes is not only a rational quantity, but equals unity.

The law just stated, which applies to all systems, indicates that in the hexagonal system $a : c$ is always an irrational quantity. If a be taken as unity, which is always done, c may be greater or less than unity, but is always an irrational quantity, and is usually given to four decimal places. As an illustration, consider the hexagonal mineral beryl, of which emerald is a variety. The ground-form of this mineral cuts two of the horizontal and the vertical axes at such distances from the origin as will make the ratio between a and c as 1 is to 0.4989 (nearly). The unit axial distances of this mineral are, then, $a = 1$ and $c = 0.4989$ (nearly). The value of c differs more or less for all hexagonal minerals. It is, then, a distinguishing characteristic of each hexagonal mineral.

Since m and n are always rational quantities (see p. 15), it follows that $na : c$ and $a : mc$ are irrational quantities; while $a : na$ is a rational quantity.

In all systems but the isometric *m* may be less than unity; and it is customary to apply this parameter (*m*) to the unit axial length of the *vertical* axis. *n* must be greater than unity in the hexagonal and isometric systems only.

First Order Position Defined.

Forms with faces whose planes cut two horizontal axes equally (at equal finite distances from the origin), and are parallel to the third horizontal axis, are said to be in the first order position.

Second Order Position Defined.

Forms with faces whose planes cut two of the horizontal axes equally and the third horizontal axis at a distance from the origin which is half that cut off on the other two horizontal axes are said to be in the second order position.

Third Order Position Defined.

Forms with faces whose planes cut all three horizontal axes unequally are said to be in the third order position.

Holohedral Hexagonal Forms Tabulated.

Name.	Symbol.	Number of faces.
1st order pyramid (Fig. 25)	$a : a : \infty a : mc$	12
1st order prism (Fig. 26)	$a : a : \infty a : \infty c$	6
2nd order pyramid (Fig. 27)	$2a : a : 2a : mc$	12
2nd order prism (Fig. 28)	$2a : a : 2a : \infty c$	6
Dihexagonal pyramid (Fig. 29)	$na : a : pa : mc$	24
Dihexagonal prism (Fig. 30)	$na : a : pa : \infty c$	12
Basal-pinacoid (Fig. 31)	$\infty a : \infty a : \infty a : c$	2

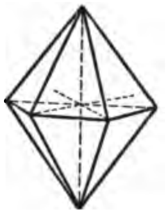


FIG. 25.—1st order pyramid.

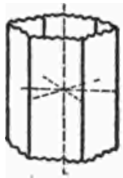


FIG. 26. — 1st order prism.

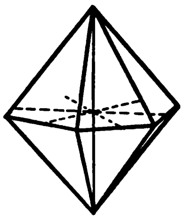


FIG. 27.—2nd order pyramid.

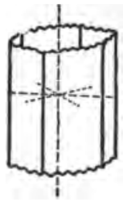


FIG. 28. — 2nd order prism.

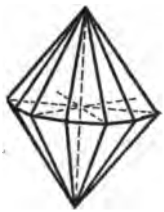


FIG. 29. — Dihexagonal pyramid.

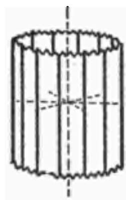


FIG. 30. — Dihexagonal prism.

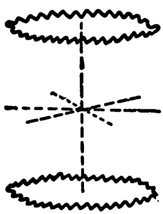


FIG. 31. — Basal pinacoid.

Synonyms for the Names of the Holohedral Hexagonal Forms.

1st order pyramid — 1st order bipyramid, or unit pyramid.
1st order prism — unit prism.
2nd order pyramid — 2nd order bipyramid.
2nd order prism — none.
Dihexagonal pyramid — dihexagonal bipyramid.
Dihexagonal prism — none.
Basal-pinacoid — basal plane.

Method of Determining Holohedral Hexagonal Forms by the Use of Symbols.

After properly orienting the crystal in the manner already described select any face and ascertain the relative distances at which its plane intersects the four crystal axes, [remembering that no face or faces extended can cut the vertical axis at the same distance from the origin as it cuts any of the horizontal axes.] If, for instance, it appears that the plane of the face selected intersects all four of the axes, but that the three horizontal axes are all cut at unequal distances from the origin, the symbol of that face (and of the form of which it is a part) is $na : a : pa : mc$. By referring to the table of holohedral hexagonal forms (which should be memorized as soon as possible) it is seen that the form is the dihexagonal pyramid. If more than one form is represented on the crystal, each may be determined in the same way.

The parameter p in the symbol of the dihexagonal pyramid and prism is not an independent variable, but is, in fact, equal to $\frac{n}{n-1}$. When n equals 3,

for instance, p will equal $\frac{3}{2}$ or $1\frac{1}{2}$. It might be better always to use $\frac{n}{n-1}$ instead of p, but, if the equality of the two symbols is always borne in mind, no confusion need result.

It is necessary to write the symbol of hexagonal forms in such a way as will make the second part of each symbol always a (or ∞a in the case of the basal-pinacoid). Thus, $na : a : pa : mc$ is correct, while $na : pa : a : mc$ is incorrect; and $a : 2a : 2a : mc$ is not the symbol of the 2nd order pyramid, while $2a : a : 2a : mc$ is the correct symbol of this form.

Suggestions for Attaining Facility in the Recognition of Forms.

Orient the crystal and determine which of the following descriptions (which should be learned at once) apply to the faces of different shape or size seen.

1st order pyramid: A face sloping down from the vertical axis directly towards the observer.

1st order prism: A vertical face extending from right to left.

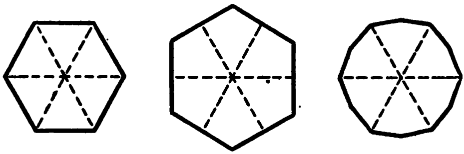
2nd order pyramid: A face sloping down from the vertical axis directly to the right or left.

A 2nd order pyramid differs in no way from a 1st order pyramid excepting in position with respect to the horizontal crystal axes; and a twelve-faced pyramid may be placed in either the 1st or 2nd order position at will. Such a pyramid may, then, be considered either a 1st or a 2nd order pyramid depending upon the set of interchangeable symmetry

axes with which the crystal axes are chosen to coincide. It is only when forms in both the 1st and 2nd order position occur on the same crystal that it is necessary to distinguish between 1st and 2nd order pyramids.

2nd order prism: A vertical face extending from front to back.

As is the case with the 2nd order pyramid, a 2nd order prism differs in no way from a 1st order prism



A B C

FIG. 32. — Diagrams showing relations of the 1st order (A), 2nd order (B), and dihexagonal (C) pyramids and prisms to the horizontal crystal axes.

excepting in position with respect to the horizontal crystal axes; and all that was said in the preceding section relative to the 2nd order pyramid applies with equal truth to the 2nd order prism.

It is customary to select the horizontal crystal axis in such a way as will place the largest and most prominent twelve-faced pyramid or six-faced prism in the 1st order position.

Pyramids and prisms intersecting in horizontal edges are always of the same order.

Dihexagonal pyramid: A face sloping down from the vertical axis in such a way that its plane intersects all three horizontal crystal axes at unequal finite distances from the origin.

Dihexagonal prism: A vertical face whose plane intersects all three horizontal crystal axes at unequal finite distances from the origin.

Basal-pinacoid: A horizontal face on top of a crystal.

Fixed and Variable Forms.

The only fixed holohedral hexagonal forms are the 1st and 2nd order prisms and the basal-pinacoid, and forms derived therefrom.

Fixed Angles of the Hexagonal System.

The only fixed angles in this system are those between the fixed forms just mentioned, namely, 90°, 120° (or 60°), and 150° (or 30°).

Miscellaneous.

The general statements made in the discussion of the holohedral division of the isometric system

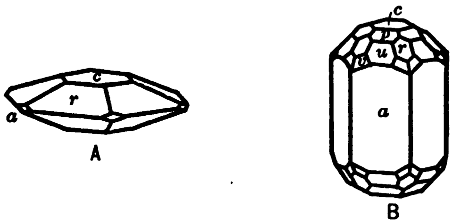


Fig. 33. — Holohedral hexagonal crystals.

A: Basal-pinacoid (*c*), 1st order pyramid (*r*), and 2nd order prism (*a*).

B: Basal-pinacoid (*c*), 1st order prism (*a*), two 1st order pyramids (*p* and *u*), 2nd order pyramid (*r*), and dihexagonal pyramid (*v*).

regarding combination of forms, determination of the number of forms, repetition of forms on a crystal,

and limiting forms applies with equal truth to all the divisions of the hexagonal system. It may be mentioned, however, that repetitions of the same variable form are very common in the hexagonal system, and crystal models showing such repeated forms are not difficult to obtain.

RHOMBOHEDRAL HEMIHEDRAL DIVISION

Development or Derivation of the Forms.

Rhombohedral hemihedral hexagonal forms may be conceived to be developed by dividing each of the holohedral forms by means of the principal symmetry plane and *the set of interchangeable secondary symmetry planes containing the crystal axes* into twelve parts, or dodecants, then suppressing all faces lying wholly within alternate parts thus obtained, and extending the remaining faces until they meet in edges or corners.

Symmetry.

Rhombohedral hemihedral forms possess three interchangeable secondary symmetry planes which intersect along a common line at angles of 60° or 120°.

Selection, Position, and Designation of the Crystal Axes.

The three directions used as crystal axes in the holohedral division are still utilized for the same purpose in the rhombohedral hemihedral division. In other words, the vertical or c axis lies at the intersection of the three secondary symmetry planes; while the three interchangeable crystallographic directions used as horizontal or a axes are so held

that one of them extends from right to left, and each of them bisects an angle between two of the secondary symmetry planes.

Orienting Crystals.

Rhombohedral hemihedral forms are oriented by holding a symmetry plane vertically *from front to back,* then rotating the crystal around the axis perpendicular to this plane until two other symmetry planes making angles of 60° to 120° with the plane first mentioned are held vertically. The crystal axes will then extend in the proper directions.

Rhombohedral Hemihedral Hexagonal Forms Tabulated.

Name.	Symbol.	Number of faces.	Form from which derived.
± Rhombohedron (Fig. 34)	$\pm\frac{a : a : \infty a : mc}{2}$	6	1st order pyramid
Hexagonal scalenohedron (Fig. 35)	$\frac{na : a : pa : mc}{2}$	12	dihexagonal pyramid
1st order prism (Fig. 26)	$\frac{a : a : \infty a : \infty c}{2}$	6	1st order prism
2nd order pyramid (Fig. 27)	$\frac{2a : a : 2a : mc}{2}$	12	2nd order pyramid
2nd order prism (Fig. 28)	$\frac{2a : a : 2a : \infty c}{2}$	6	2nd order prism
Dihexagonal prism (Fig. 30)	$\frac{na : a : pa : \infty c}{2}$	12	dihexagonal prism
Basal-pinacoid (Fig. 31)	$\frac{\infty a : \infty a : \infty a : c}{2}$	2	basal-pinacoid

Synonyms for the Names of the Rhombohedral Hemihedral Hexagonal Forms.

Rhombohedron — none.

Scalenohedron — none.

Positive and Negative Forms Distinguished.

All those forms produced by the suppression of faces lying within the *same set* of alternating dodecants are said to be of the same sign (+ or −). It is customary to consider those forms with faces

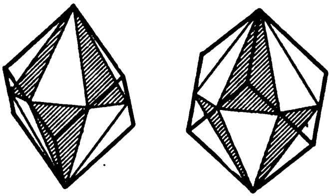


FIG. 34. — Positive (on left) and negative (on right) rhombohedrons containing the forms from which they are derived. The suppressed faces are shaded.

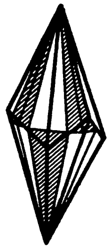


FIG. 35. — Hexagonal scalenohedron containing the form from which it is derived. The suppressed faces are shaded.

largely or entirely included within the upper dodecant directly facing the observer as +, while those with faces in the upper dodecant at the back of the crystal furthest from the observer are −. In reality a + rhombohedron differs in no way from a − rhombohedron excepting in position; and a rhombohedron may be held in either the + or − position at will. It is customary to hold a crystal in such a way as to bring the largest and most prominent rhombohedron face principally or entirely into the upper dodecant facing the observer, which will make this form a + one. It is possible, but unnecessary, to distinguish between + and − hexagonal scalenohedrons.

If the sign of a form is not specifically stated as being −, it is always assumed that the form is +.

The forms on a crystal may all be of the same sign, or + and − forms may be combined.

Method of Determining Rhombohedral Hemihedral Hexagonal Forms by the Use of Symbols.

After properly orienting the crystal all the forms but the rhombohedron and scalenohedron may be easily identified by applying the rules already given for the determination of holohedral forms of the same name. The two hemihedral forms new in shape may be recognized by determining the symbol of any face in the manner described in the discussion of holohedral forms, dividing the symbol by 2, and then ascertaining from the table the name of the form possessing this symbol.

Suggestions for Attaining Facility in the Recognition of Forms.

Orient the crystal and determine which of the following descriptions (which should be learned at once) apply to the face or faces of different shape or size seen. It is assumed that the student is already familiar with the rules for recognizing those forms identical in shape with the holohedral ones (see p. 51).

\+ *Rhombohedron:* A face sloping down from the vertical axis directly toward the observer. A rhombohedron has three faces at each end of the vertical axis so arranged that a face on top is directly above an edge below.

− *Rhombohedron:* A face sloping down from the vertical axis directly away from the observer, at the back of the crystal.

Hexagonal scalenohedron: A face sloping down from the vertical axis in such a way that its plane intersects all three horizontal crystal axes at unequal finite distances from the origin.

The hexagonal scalenohedron is most readily confused with the 2nd order pyramid. To distinguish them, it should be remembered that the upper and lower faces of the latter always intersect in horizontal edges, and that the interfacial angles of the 2nd order pyramid, measured across edges converging towards the vertical axis, are all equal. Neither statement is true as regards the hexagonal scalenohedron.

Rules and Conventions Relating to Rhombohedrons.

As may be gathered from the statements already made in this volume (see p. 13), the unit rhombohedron in the case of any given mineral species is usually the rhombohedron occurring most commonly on crystals of that mineral. In the case of rhombohedral minerals with a well-developed rhombohedral cleavage (see p. 142), however, it is sometimes found more convenient to select the cleavage rhombohedron as the unit rhombohedron, and to call all distances at which this unit rhombohedron cuts the a and c axes the unit axial distances a and c. a is made equal to unity, and c is then some irrational quantity either greater or less than unity. It is customary to designate the unit rhombohedron by the symbol R which may be + or − according to its position on the crystal.

All other rhombohedrons than R will cut the a and c axes at such distances that if a is made equal

to unity, mc will be some rational multiple of c. If m is equal to 2, the rhombohedron is represented by the symbol 2 R, either + or −; while if m is equal to ½, the rhombohedron is represented by the symbol ½ R, either + or −. Similarly, a rhombohedron intersecting the vertical axis at $3c$ may be represented by the symbol 3 R, etc.

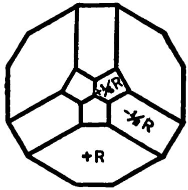
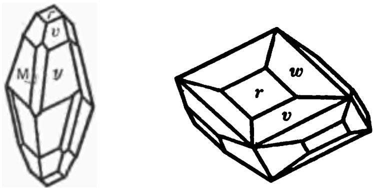


FIG. 36. — View from above of a unit rhombohedron, with its edges truncated by − ½ *R*, and the edges of the latter truncated by + ¼ *R*.

It may be readily proved geometrically (although to offer such a proof is beyond the scope of this work) that any rhombohedron which truncates the edges of another rhombohedron will intersect the vertical axis at one-half the c value of the truncated rhombohedron and will be of opposite sign. In other words, + R may have its edges truncated by − ½ R; − ½ R may have its edges truncated by + ¼ R, etc. Stated differently, − ½ R will truncate the edges of + R; or − 2 R will truncate the edges of + 4 R.

From what has been said it is evident that a + rhombohedron always truncates a − rhombohedron or vice versa. In order to ascertain whether one rhombohedron is truncating another, it is only necessary to determine whether one rhombohedron intersects the other in parallel edges. If a rhombohedron is intersected by other rhombohedron faces of opposite sign so as to produce parallel edges, the former is truncating the latter. Fig. 36 illustrates a crystal viewed from above that shows + R, − ½ R,

and $+\frac{1}{4}$ R. If the rhombohedron represented in the case just mentioned as $-\frac{1}{2}$ R be taken as the unit rhombohedron, + R, the other rhombohedrons shown will be -2 R and $-\frac{1}{2}$ R, respectively.



A B

FIG. 37. — Rhombohedral hemihedral hexagonal crystals.

A: Two + rhombohedrons (*r* and *M*) and two scalenohedrons (*y* and *v*).

B: + rhombohedron (*r*) and two scalenohedrons (*w* and *v*).

PYRAMIDAL HEMIHEDRAL DIVISION

Development or Derivation of the Forms.

Pyramidal hemihedral hexagonal forms may be conceived to be developed by dividing each of the holohedral forms by means of all six secondary symmetry planes into twelve parts, then suppressing all faces lying wholly within alternate parts thus obtained, and extending the remaining faces until they meet in edges or corners.

Symmetry.

Pyramidal hemihedral forms possess only one symmetry plane which is in the position of the principal symmetry plane existing in the holohedral division. It is, however, in the pyramidal hemihedral division a secondary rather than a principal

symmetry plane since there are no interchangeable symmetry planes perpendicular to it.

Pyramidal hemihedral hexagonal forms are, then, characterized by the presence of one secondary symmetry plane, and a general six-fold arrangement of faces.

Selection, Position, and Designation of the Crystal Axes.

The vertical or *c* crystal axis is made to coincide with the secondary symmetry axis. Three interchangeable horizontal axes parallel to prominent crystallographic directions at angles of 60° or 120° to each other are also selected, and one of these is so placed as to extend from right to left. Being interchangeable, all are called *a* axes.

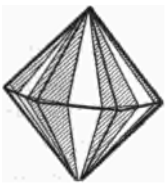


FIG. 38. — 3rd order pyramid containing the form from which it is derived. Suppressed faces are shaded.

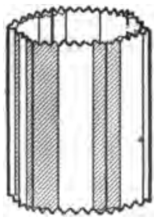


FIG. 39. — 3rd order prism containing the form from which it is derived. Suppressed faces are shaded.

Orienting Crystals.

The secondary symmetry plane is held horizontally. The crystal is then rotated around the symmetry axis until the most prominent pyramid or prism lies in the first order position. The crystal axes will then extend in the proper directions.

Pyramidal Hemihedral Hexagonal Forms Tabulated.

Name.	Symbol.	Number of faces.	Form from which derived.
3rd order pyramid (Fig. 38).	$\frac{na:a:pa:mc}{2}$	12	dihexagonal pyramid
3rd order prism (Fig. 39)....	$\frac{na:a:pa:\infty c}{2}$	6	dihexagonal prism
1st order pyramid (Fig. 25).	$\frac{a:a:\infty a:mc}{2}$	12	1st order pyramid
1st order prism (Fig. 26)....	$\frac{a:a:\infty a:\infty c}{2}$	6	1st order prism
2nd order pyramid (Fig. 27)	$\frac{2a:a:2a:mc}{2}$	12	2nd order pyramid
2nd order prism (Fig. 28)...	$\frac{2a:a:2a:\infty c}{2}$	6	2nd order prism
Basal-pinacoid (Fig. 31).....	$\frac{\infty a:\infty a:\infty a:c}{2}$	2	basal-pinacoid

Synonyms for the Names of the Pyramidal Hemihedral Hexagonal Forms.

3rd order pyramid—3rd order bipyramid.

3rd order prism—none.

Method of Determining Pyramidal Hemihedral Hexagonal Forms by the Use of Symbols.

After properly orienting the crystal in the manner already described all the forms but the 3rd order pyramid and prism may be identified easily by applying the rules already given for the determination of holohedral forms of the same name. 3rd order pyramids and prisms may be recognized by determining the symbol of any face in the manner already described in the discussion of holohedral forms, dividing this symbol by 2, and then ascertaining from the table the name of the form possessing this symbol.

Suggestions for Attaining Facility in the Recognition of Forms.

Orient the crystal and determine which of the following descriptions (which should be learned at once) apply to the face or faces of different shape or size seen. It is possible, but unnecessary, to distinguish between + and − forms in this division. It is assumed that the student is already familiar with the rules for recognizing those forms identical in shape and position with the holohedral ones (see p. 51).

3rd order pyramid: A face sloping down from the vertical axis so that its plane intersects all three horizontal crystal axes at unequal finite distances from the origin.

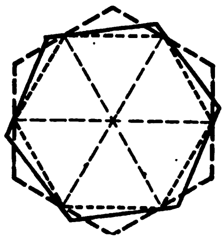


FIG. 40. — Diagram showing the relations of the 1st order (dotted lines), 2nd order (broken lines), and 3rd order (solid lines) pyramids and prisms to the horizontal crystal axes.

The 3rd order pyramid differs in no way from the 1st or 2nd order pyramid excepting in position with respect to the horizontal crystal axes. All three types of 12-faced pyramids may have the same appearance; and any such pyramid may be held at will as a 1st, 2nd, or 3rd order pyramid. The 3rd order pyramid is skewed or twisted through a small angle (less than 30°) either to the right or left away from the position of the 1st or 2nd order pyramid. Fig. 40 shows how the horizontal axes are cut by 1st, 2nd, and 3rd order pyramids and prisms.

3rd order prism: A vertical face whose plane inter-

sects the three horizontal crystal axes at unequal finite distances from the origin.

All that was said in the preceding section relative to the 3rd order pyramid applies with equal truth to the 3rd order prism.

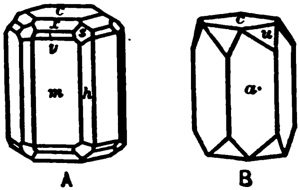


FIG. 41. — Pyramidal hemihedral hexagonal crystals.

A: Basal-pinacoid (*c*), 1st order prism (*m*), two 1st order pyramids (*x* and *y*), 2nd order pyramid (*s*), and 3rd order prism (*h*).

B: Basal-pinacoid (*c*), 2nd order prism (*a*), and 3rd order pyramid (*u*).

TRIGONAL HEMIHEDRAL DIVISION

Development or Derivation of the Forms.

Trigonal hemihedral hexagonal forms may be conceived to be developed by dividing each of the holohedral forms by means of the three secondary symmetry planes containing the horizontal crystal axes into six parts, then suppressing all faces lying wholly within alternate parts thus obtained, and extending the remaining faces until they meet in edges or corners.

Symmetry.

The trigonal hemihedral division of the hexagonal system is characterized by the presence of one principal and three interchangeable secondary symmetry planes which lie at right angles to the principal

symmetry plane. The interchangeable symmetry planes make angles of 60° or 120° with each other.

Selection, Position, and Designation of the Crystal Axes.

The principal symmetry axis is chosen as the vertical or *c* crystal axis; while three interchangeable horizontal directions, each of which bisects the angle between two secondary symmetry planes, constitute the horizontal or *a* axes. One of these is so held as to extend from right to left.

Orienting Crystals.

Trigonal hemihedral forms are oriented by holding the principal symmetry plane horizontally, and one secondary symmetry plane vertically and extending from front to back. The crystal axes will then extend in the proper direction.

Trigonal Hemihedral Hexagonal Forms Tabulated.

Name.	Symbol.	Number of faces.	Form from which derived.
± 1st order trigonal pyramid (Fig. 42).........	$\pm\frac{a:a:\infty a:mc}{2}$	6	1st order pyramid
± 1st order trigonal prism (Fig. 43)..............	$\pm\frac{a:a:\infty a:\infty c}{2}$	3	1st order prism
Ditrigonal pyramid (Fig. 44)...................	$\frac{na:a:pa:mc}{2}$	12	dihexagonal pyramid
Ditrigonal prism (Fig. 45)..	$\frac{na:a:pa:\infty c}{2}$	6	dihexagonal prism
2nd order pyramid (Fig. 27)	$\frac{2a:a:2a:mc}{2}$	12	2nd order pyramid
2nd order prism (Fig. 28)...	$\frac{2a:a:2a:\infty c}{2}$	6	2nd order prism
Basal-pinacoid (Fig. 31).....	$\frac{\infty a:\infty a:\infty a:c}{2}$	2	basal-pinacoid

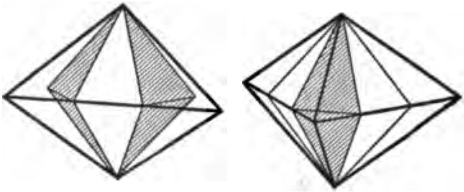


FIG. 42. — Positive (on left) and negative (on right) 1st order trigonal pyramids containing the forms from which they are derived. The suppressed faces are shaded.

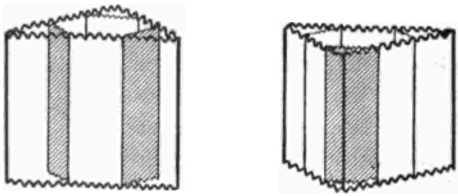


FIG. 43. — Positive (on left) and negative (on right) 1st order trigonal prisms containing the forms from which they are derived. The suppressed faces are shaded.

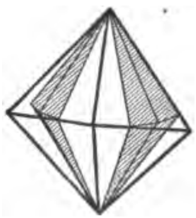


FIG. 44. — Ditrigonal pyramid containing the form from which it is derived. The suppressed faces are shaded.

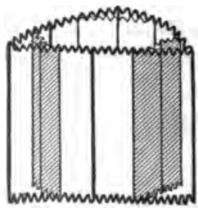


FIG. 45.— Ditrigonal prism containing the form from which it is derived. The suppressed faces are shaded.

Synonyms for the Names of the Trigonal Hemihedral Hexagonal Forms.

1st order trigonal pyramid — trigonal bipyramid of the 1st order.

Ditrigonal pyramid — ditrigonal bipyramid.

Positive and Negative Forms Distinguished.

All those forms produced by the suppression of faces lying within the same set of alternating dodecants are said to be of the same sign (+ or −). It is possible, but unnecessary, to distinguish between + and − ditrigonal pyramids and prisms. It is customary to consider a trigonal prism or pyramid with a face or faces extending from right to left between the vertical axis and the observer as +, while one with such a face or faces back of the vertical axis is −. In reality, a + trigonal pyramid differs in no way from a − trigonal pyramid excepting in position; and a trigonal pyramid may be held in either the + or − position at will. The same statements hold as regards the trigonal prism. Convention requires that the largest and most prominent trigonal pyramid or prism should be held in such a way as to bring it into the + position.

All the other statements regarding + and − forms made in the discussion of the rhombohedral hemihedral division (see p. 57) apply with equal truth to the division under consideration.

Method of Determining Trigonal Hemihedral Hexagonal Forms by the Use of Symbols.

After properly orienting the crystal in the manner already described the 2nd order pyramid and prism

and the basal-pinacoid may be identified easily by applying the rules already given for the determination of holohedral forms of the same name.

The four hemihedral forms differing in shape from the holohedral ones from which they were derived may be recognized by determining the symbol of any face in the manner described in the discussion of holohedral forms, dividing this symbol by 2, and then ascertaining from the table the name of the form possessing this symbol.

Suggestions for Attaining Facility in the Recognition of Forms.

Orient the crystal and determine which of the following descriptions (which should be learned at once) apply to the face or faces of different shape or size seen. It is assumed that the student is already familiar with the rules for recognizing those forms identical in shape with the holohedral ones (see p. 51).

+ *1st order trigonal pyramid:* A face sloping down from the vertical axis directly toward the observer. This face occupies exactly the same position as that of a + rhombohedron. However, a 1st order trigonal pyramid differs from a rhombohedron in that the three faces at one end of the vertical axis intersect those at the other end in edges which are horizontal.

— *1st order trigonal pyramid:* A face sloping down from the vertical axis directly away from the observer at the back of the crystal. This face occupies exactly the same position as that of a — rhombohedron.

+ *1st order trigonal prism:* A vertical face extending directly from right to left between the vertical axis and the observer.

– *1st order trigonal prism:* A vertical face extending directly from right to left at the back of the crystal.

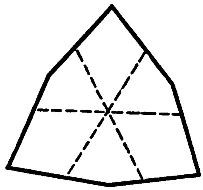


FIG. 46. — Diagram showing the relation of the faces of the trigonal hemihedral ditrigonal pyramid and prism to the horizontal crystal axes. (Compare with Fig. 54.)

Ditrigonal pyramid: A face sloping down from the vertical axis in such a way that its plane intersects the three horizontal crystal axes at unequal finite distances from the origin. The six faces at each end of the vertical axis occupy exactly the same positions as the six faces making up half of a scalenohedron, but may be distinguished from scalenohedron faces by the fact that the faces at opposite ends of the vertical axis intersect in edges that are horizontal.

Ditrigonal prism: A vertical face whose plane intersects all three horizontal crystal axes at unequal finite distances from the origin.

Hemimorphism.

A hemimorphic crystal, as already stated (p. 17), is one in which the law of axes (see p. 16) is violated so far as one crystal axis is concerned. In other words, on a hemimorphic crystal the opposite ends of one crystal axis are not cut by the same number of similarly placed faces. For instance, there may be one or more pyramids on one end of a crystal axis,

and only a basal pinacoid on the other; or the forms at both ends of an axis may have the same names, but different slopes.

Theoretically, hemimorphic forms may occur in all divisions of the hexagonal system, but they are relatively unimportant on any kind of crystals already discussed excepting trigonal hemihedral hexagonal ones.

Naming Hemimorphic Forms.

It is customary to hold the axis whose ends are treated differently vertically. After properly orienting the crystal the forms on the upper end of the crystal are given first, then the crystal is turned upside down, and those on the other end are named. Forms common to both ends, like prisms and pinacoids (other than the basal pinacoid), are mentioned but once.

In writing out the names of the forms on a hemimorphic crystal it is customary to separate the names of the forms on the differing ends of the crystal by means of a horizontal line.

Importance of Hemimorphism in the Trigonal Hemihedral Hexagonal Division.

With one possible exception, all natural minerals crystallizing in this division of the hexagonal system are hemimorphic, that is, the opposite ends of their vertical axes are not intersected by the same number of similar faces similarly placed. This eliminates the principal symmetry plane, and gives the trigonal and ditrigonal pyramids the appearance of rhombohedrons and scalenohedrons. That the crystals

resulting are not rhombohedral is, however, usually shown plainly by the presence of a prominent trigonal prism, a form which does not occur in the rhombohedral hemihedral division. This gives trig-

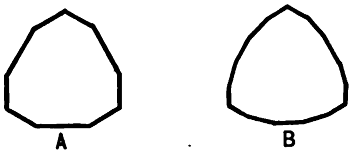


FIG. 47. — Typical horizontal sections of the trigonal hemihedral mineral tourmaline.

onal hemihedral crystals cross-sections which are either triangular or (more commonly) spherical-triangular. Fig. 47 shows two typical cross-sections of the mineral tourmaline which is the commonest species crystallizing in this division of the system.

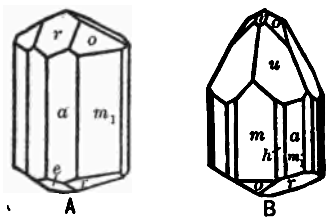


FIG. 48. — Trigonal hemihedral hexagonal crystals (hemimorphic).

A: + and −1st order trigonal pyramids (*r* and *o*), −1st order trigonal prism (m_1), and 2nd order prism (*a*). On other end: + and −1st order trigonal pyramids (*r* and *e*).

B: + and −1st order trigonal pyramids (*r* and *o*), + and −1st order trigonal prisms (*m* and m_1), ditrigonal pyramid (*u*), ditrigonal prism (*h*), and 2nd order prism (*a*). On other end: + and −1st order trigonal pyramids (*r* and *o*).

TRAPEZOHEDRAL HEMIHEDRAL DIVISION

Trapezohedral hemihedral forms may be conceived to be developed by dividing each holohedral form by the principal and all the secondary symmetry planes into 24 parts, then suppressing all faces lying wholly within alternate parts thus obtained, and extending all the remaining faces until they meet in edges or corners.

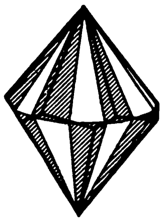


FIG. 49. — Hexagonal trapezohedron containing the form from which it is derived. The suppressed faces are shaded.

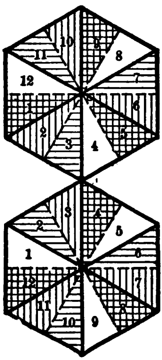
As the dihexagonal pyramid is the only hexagonal form with 24 faces, it is evident that a dihexagonal pyramid face is the only one that can lie wholly within one of the parts obtained by dividing a hexagonal crystal in the manner just specified. The dihexagonal pyramid is, then, the only hexagonal form from which a trapezohedral hemihedral form differing from the holohedral one in shape and name can be derived. This new form is called the hexagonal trapezohedron (Fig. 49). This form may be either right or left-handed, but, since no mineral is known to crystallize in this division, its further discussion seems unnecessary.

TRAPEZOHEDRAL TETARTOHEDRAL DIVISION

Development or Derivation of the Forms.

Trapezohedral tetartohedral hexagonal forms may be conceived to be developed by the simultaneous application of the rhombohedral and trapezohedral hemihedrisms. In other words, a holohedral form is first divided into dodecants by means of the principal symmetry plane and the three secondary symmetry planes containing the crystal axes, as in the development of rhombohedral hemihedral forms; and faces or portions of faces lying within alter-

nating dodecants are marked tentatively as subject to suppression. The holohedral form is then divided by means of the principal and all of the secondary symmetry planes into twenty-four parts, as in the development of trapezohedral hemihedral forms; and faces or portions of faces lying within alternating parts thus obtained are marked tentatively as subject to suppression. If, after this has been done, it is found that any crystal face has been marked in such a way as to indicate that all portions of it are tentatively subject to suppression, that



Upper six faces as viewed from above.

Lower six faces as viewed from below.

FIG. 50.—Diagram to illustrate the development of the 2nd order trigonal pyramid from the 2nd order pyramid as explained in the text.

crystal face is suppressed; but, if all or any portion of a crystal face remains unmarked as subject to suppression, that face is extended until it meets other similar faces in edges or corners.

As an illustration of the process just outlined, consider a 2nd order pyramid (see Fig. 50). If this form is divided by means of the principal symmetry

plane and the secondary symmetry planes containing the crystal axes, and faces or parts of faces lying within alternating dodecants thus obtained are marked tentatively as subject to suppression, parts of faces 1 and 2, 5 and 6, and 9 and 10 on top of the crystal; and 3 and 4, 7 and 8, and 11 and 12 on the other end should be so marked, as indicated by the vertically hatched portions on Fig. 50. If, then, the form be divided by means of the principal symmetry plane and all six secondary symmetry planes into twenty-four parts, and faces or parts of faces lying within alternating parts thus obtained be marked tentatively as subject to suppression, the parts of the faces so marked will be those numbered 1, 3, 5, 7, 9, and 11 on top of the crystal, and 2, 4, 6, 8, 10, and 12 on the bottom, as indicated by the horizontally hatched portions in Fig. 50. This leaves the half-faces 4, 8, and 12, on top, and the half-faces 1, 5, and 9, on the other end of the crystal unhatched, while all the remaining faces are completely hatched. If, then, we extend the faces partially unhatched until they meet in edges or corners as illustrated by Fig. 51, we shall obtain the trapezohedral tetartohedral derivative of the second order pyramid, namely, the 2nd order trigonal pyramid. By applying this method to the other holohedral forms, their trapezohedral tetartohedral derivatives may be ascertained.

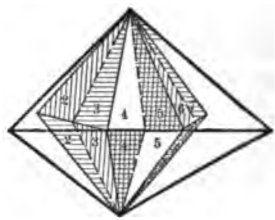


FIG. 51. — 2nd order trigonal pyramid containing the form from which it is derived. Suppressed faces are shaded.

Symmetry.

Trapezohedral tetartohedral forms possess no symmetry planes whatever, but show a three-fold or six-fold arrangement of faces.

Selection, Position, and Designation of the Crystal Axes.

The four directions used as crystal axes in the holohedral division are still utilized for the same purpose in the trapezohedral tetartohedral division. In other words, one vertical or *c* axis and three interchangeable horizontal or *a* axes intersecting at angles of 60° or 120° are utilized. One of the latter is held from right to left.

Orienting Crystals.

The absence of all symmetry planes, and the presence of two sets of prominent interchangeable crystallographic directions which may be so placed as to occupy the position of the horizontal crystal axes makes it impossible to give any rules for orienting trapezohedral tetartohedral crystals based entirely on symmetry planes or crystallographic directions.

Since quartz is the only mineral which occurs at all commonly in recognizable trapezohedral tetartohedral crystals, it seems best to suggest rules for orientation applicable especially to that mineral. These are as follows:

On most crystals one crystallographic direction emerging on the surface of the crystal at a corner formed by the intersection of three or six faces making equal angles with each other is usually very

prominent. This is selected as the vertical or c axis, and is not interchangeable with any other crystallographic direction. The crystal is then rotated around the vertical axis until prominent prism faces occupy the 1st order position; or, if a prominent prism is lacking, prominent pyramidal faces are placed in the 1st order position. The three interchangeable horizontal crystal axes will then extend in the proper directions.

Trapezohedral Tetartohedral Hexagonal Forms Tabulated.

Name.	Symbol.	Number of faces.	Form from which derived.
±Rhombohedron (Fig. 34).	$\pm\frac{a : a : \infty a : mc}{4}$	6	1st order pyramid
±2nd order trigonal pyramid (Fig. 51)..........	$\pm\frac{2a : a : 2a : mc}{4}$	6	2nd order pyramid
±2nd order trigonal prism (Fig. 52)...............	$\pm\frac{2a : a : 2a : \infty c}{4}$	3	2nd order prism
Trigonal trapezohedron (Fig. 53)..............	$\frac{na : a : pa : mc}{4}$	6	dihexagonal pyramid
Ditrigonal prism (Fig. 54)...	$\frac{na : a : pa : \infty c}{4}$	6	dihexagonal prism
1st order prism (Fig. 26)....	$\frac{a : a : \infty a : \infty c}{4}$	6	1st order prism
Basal-pinacoid (Fig. 31).....	$\frac{\infty a : \infty a : \infty a : c}{4}$	2	basal-pinacoid

Synonyms for the Names of the Trapezohedral Tetartohedral Hexagonal Forms.

2nd order trigonal pyramid — trigonal bipyramid of the 2nd order.

2nd order trigonal prism — unsymmetrical trigonal prism.

Trigonal trapezohedron — quadrilateral trapezohedron.

Positive and Negative Forms Distinguished.

It is possible to distinguish between + and − variations of each of the trapezohedral tetartohedral forms which differ in shape from the holohedral ones

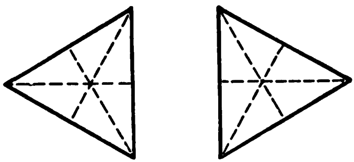


FIG. 52. — Diagrams showing relations of the positive (on left) and negative (on right) 2nd order trigonal pyramid and prism to the horizontal crystal axes.

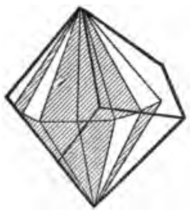


FIG. 53. — Trigonal trapezohedron containing the form from which it is derived. The suppressed faces are shaded.

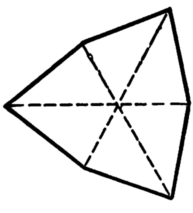


FIG. 54. — Diagram showing the relation of the tetartohedral ditrigonal prism to the horizontal crystal axes.

from which they are derived. It is, however, unnecessary to differentiate between + and − trigonal trapezohedrons and ditrigonal prisms.

+ and − rhombohedrons are distinguished in exactly the same way as are + and − rhombohe-

drons in the rhombohedral hemihedral division (see p. 56).

It is customary to call those trigonal prisms and pyramids + which have a face or faces extending directly from front to back at the *right* of the vertical axes; while those with a similar face or faces at the *left* of the vertical axis are called −. See Fig. 52.

Symbols of Tetartohedral Forms.

The symbol of a tetartohedral form in any system is the same as that of the holohedral form from which it is derived excepting that it is written as a fraction with the figure 4 as the denominator. This does not mean that, in the case of the tetartohedral forms, the axes are intersected at one-fourth the holohedral axial lengths, but is merely a conventional method of indicating that the symbol is that of a quarter (tetartohedral) form. The symbol $\frac{na : a : pa : mc}{4}$ is read *na, a, pa, mc* over 4.

Method of Determining Trapezohedral Tetartohedral Hexagonal Forms by the Use of Symbols.

After properly orienting the crystal the 1st order prism and basal-pinacoid may be identified easily by applying the rules already given for the determination of holohedral forms of the same name. The tetartohedral forms new in shape may be recognized by determining the symbol of any face in the manner described in the discussion of holohedral forms, dividing this symbol by 4, and then ascertaining from the table the name of the form possessing this symbol.

Suggestions for Attaining Facility in the Recognition of Forms.

Orient the crystal and determine which of the following descriptions (which should be learned at once) apply to the face or faces of different shape or size seen. It is assumed that the student is already familiar with the rules for recognizing those forms identical in shape with the holohedral ones (see p. 51).

$\pm$ *Rhombohedron:* Since the trapezohedral tetartohedral rhombohedrons differ in no way excepting in internal molecular arrangement from the rhombohedral hemihedral forms of the same name, rules for the recognition of the + and − rhombohedrons given in the description of rhombohedral hemihedral forms (see p. 57) may be used in identifying such forms in this division.

+ *2nd order trigonal pyramid:* A face which slopes down from the vertical axis directly to the right.

The 2nd order trigonal pyramid does not differ in appearance from the 1st order trigonal pyramid occurring in the trigonal hemihedral division, but does differ from the latter in its position with reference to the horizontal crystal axes.

− *2nd order trigonal pyramid:* A face which slopes down from the vertical axis directly to the left.

+ *2nd order trigonal prism:* A vertical face extending from front to back at the right of the vertical axis.

The 2nd order trigonal prism does not differ in appearance from the 1st order trigonal prism occurring in the trigonal hemihedral division, but does

differ from the latter in its position with reference to the horizontal crystal axes.

— *2nd order trigonal prism:* A vertical face extending from front to back at the left of the vertical axis.

Trigonal trapezohedron: A face sloping down from the vertical axis in such a way that its plane intersects all three horizontal crystal axes at unequal finite distances from the origin.

It is possible, but not necessary, to distinguish between right- and left-handed trigonal trapezohedrons.

The trigonal trapezohedron, like the 1st and 2nd order trigonal pyramids and the rhombohedron, has three faces at each end of the vertical axis, but the faces on top do not intersect those below in horizontal edges, as is the case with the trigonal pyramid; nor is a face on top directly above an edge below, as is the case with the rhombohedron. The three faces on one end appear, in fact, to have been twisted around the vertical axis through a small angle (less than 30°) to the right or left, placing them in an unsymmetrical position with reference to those at the other end of the crystal.

The trigonal trapezohedron is most apt to be confused with a 2nd order trigonal pyramid. These may usually be distinguished easily if the following tests are applied:

If a vertical plane is passed through an edge between two equally steep 1st order faces, it will bisect the angle between two diverging edges of a 2nd order trigonal pyramid face directly above or below the edge first mentioned. Equally steep 1st order faces in this division of the system are those of the 1st

order prism, and, in the case of quartz, those between the most prominent + and − rhombohedrons.

The statement just made will not be found true where trigonal trapezohedrons lie over or under the edges formed by the intersection of equally steep 1st order faces.

Ditrigonal prism: The tetartohedral ditrigonal prism closely resembles the trigonal hemihedral form of the same name (see p. 66), but differs therefrom in that it appears to have a symmetry plane running from right to left (see Fig. 54), while the hemihedral form has a symmetry plane extending from front to back (see Fig. 46).

General Observations.

It has already been mentioned that quartz is the only common mineral crystallizing in this division of the hexagonal system, and it should be noted that the crystallization of quartz is peculiar in that the most prominent faces are a 1st order prism (sometimes missing) and an equally or unequally developed + and − rhombohedron the faces of which make equal angles with the prism faces. In the absence of other forms, the combination last mentioned appears to be rhombohedral hemihedral, while the first combination appears to be holohedral. The presence of either a trigonal trapezohedron or trigonal pyramid is sufficient to prove the crystal tetartohedral, but these forms are very rare. The trigonal and ditrigonal prisms and basal-pinacoid are almost never found on quartz crystals.

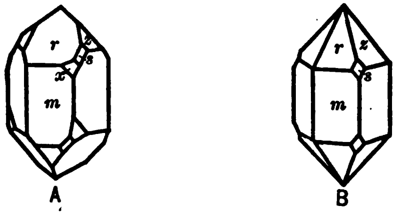


Fig. 55. — Trapezohedral tetartohedral hexagonal crystals.

A: + and − rhombohedrons (*r* and *z*), 1st order prism (*m*), − 2nd order trigonal pyramid (*s*), and trigonal trapezohedron (*x*).

B: + and − rhombohedron (*r* and *z*) equally developed, 1st order prism (*m*), and − 2nd order trigonal pyramid (*s*).

Application of the Law Governing Combination of Forms.

It has already been mentioned that one of the commonest mistakes made in determining crystal forms is the naming of two or more forms which cannot possibly occur on the same crystal since their presence would be in direct violation of the law governing the combination of forms (see p. 36). The student should thoroughly familiarize himself with the table on p. 83 if he wishes to avoid the mistake mentioned.

Inspection of this table will show that the basal-pinacoid is the only form that occurs unchanged in all five of the divisions tabulated. It is, then, the only form that can be combined with any other of the forms in the divisions considered. Further, it should be noticed that the 1st order prism occurs unchanged in all divisions but the trigonal hemihedral, while the 2nd order pyramid and prism occur unchanged in all divisions but the trapezohedral tetartohedral. The forms in the 3rd

Holohedral forms.	Rhombohedral hemihedral forms.	Pyramidal hemihedral forms.	Trigonal hemihedral forms.	Trapezohedral tetartohedral forms.
1st order pyramid......	*rhombohedron*	1st order pyramid	*1st order trigonal pyramid*	*rhombohedron*
1st order prism.........	1st order prism	1st order prism	*1st order trigonal prism*	1st order prism
2nd order pyramid.....	2nd order pyramid	2nd order pyramid	2nd order pyramid	*2nd order trigonal pyramid*
2nd order prism........	2nd order prism	2nd order prism	2nd order prism	*2nd order trigonal prism*
Dihexagonal pyramid..	*hexagonal scalenohedron*	*3rd order pyramid*	*ditrigonal pyramid*	*trigonal trapezohedron*
Dihexagonal prism.....	dihexagonal prism	*3rd order prism*	*ditrigonal prism*	*ditrigonal prism*
Basal-pinacoid.........	basal-pinacoid	basal-pinacoid	basal-pinacoid	basal-pinacoid

order position are apt to give the most trouble, since the dihexagonal pyramid yields a form of new name and shape in each of the hemihedral and tetartohedral divisions; while the dihexagonal prism yields a form of new name and shape in all divisions but the rhombohedral hemihedral.

RHOMBOHEDRAL TETARTOHEDRAL DIVISION

Rhombohedral tetartohedral forms may be conceived to be developed by the simultaneous application of the rhombohedral and pyramidal hemihedrisms, according to the principles outlined in the discussion of the trapezohedral tetartohedral hexagonal division (see p. 72). The names of the resulting rhombohedral tetartohedral forms together with their symbols, number of faces, and the name of the corresponding holohedral forms are shown on the following table:

Name.	Symbol.	Number of faces.	Form from which derived.
1st order rhombohedron..	$\frac{a : a : \infty a : mc}{4}$	6	1st order pyramid
2nd order rhombohedron.	$\frac{2a : a : 2a : mc}{4}$	6	2nd order pyramid
3rd order rhombohedron..	$\frac{na : a : pa : mc}{4}$	6	dihexagonal pyramid
3rd order prism...........	$\frac{na : a : pa : \infty c}{4}$	6	dihexagonal prism
1st order prism...........	$\frac{a : a : \infty a : \infty c}{4}$	6	1st order prism
2nd order prism..........	$\frac{2a : a : 2a : \infty c}{4}$	6	2nd order prism
Basal-pinacoid...........	$\frac{\infty a : \infty a : \infty a : c}{4}$	2	basal-pinacoid

The rhombohedral tetartohedral 3rd order prism is exactly like the form of the same name in the pyramidal hemihedral

division (see p. 63). The 1st order rhombohedron is exactly like the rhombohedron occurring in the rhombohedral hemihedral and the trapezohedral tetartohedral divisions; while the 2nd and 3rd order rhombohedrons differ from the 1st order form of the same name only in position with reference to the crystal axes. The former is, of course, in the 2nd order position, while the latter is in the 3rd order position.

Further consideration of this division seems unnecessary since few minerals are rhombohedral tetartohedral, and these are comparatively rare.

Table of Hexagonal Symbols Used by Various Authorities.

	Weiss.	Naumann.	Dana.	Miller.
1st order pyramid	$a : a : \infty a : mc$	mP	m	$(h0\bar{h}i)$
1st order prism	$a : a : \infty a : \infty c$	∞ P	I	$(10\bar{1}0)$
2nd order pyramid	$2a : a : 2a : mc$	mP 2	m–2	$(hh\,2h\,2i)$
2nd order prism	$2a : a : 2a : \infty c$	∞ P 2	i–2	$(10\bar{2}0)$
Dihexagonal pyramid	$na : a : pa : mc$	mPn	m–n	$(hk\bar{l}i)$
Dihexagonal prism	$na : a : pa : \infty c$	∞ Pn	i–n	$(hk\bar{l}0)$
Basal-pinacoid	$\infty a : \infty a : \infty a : c$	0 P	O	(0001)

Weiss, Naumann, and Dana divide the holohedral symbols by 2 and by 4 when referring to hemihedral and tetartohedral forms, respectively. Miller prefixes various Greek letters when forming the symbols of hemihedral and tetartohedral forms.

CHAPTER IV

TETRAGONAL SYSTEM

HOLOHEDRAL DIVISION

Symmetry.

The holohedral division of the tetragonal system is characterized by the presence of one principal and four secondary symmetry planes which lie at right angles to the principal symmetry plane. The secondary symmetry planes are arranged in two pairs. The planes of each pair intersect each other at an angle of 90° and are interchangeable; while the planes of one pair are non-interchangeable with those of the other pair which they intersect at an angle of 45°.

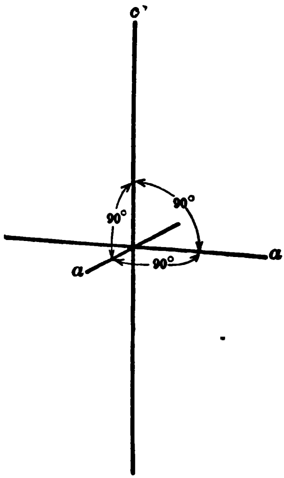


FIG. 56. — Crystal axes of the tetragonal system.

Selection, Position, and Designation of the Crystal Axes.

The principal symmetry axis is chosen as one of the crystal axes, is held vertically, and is called the *c* axis. Two other crystal axes are so selected as to coincide with one set of interchangeable secondary symmetry axes. One is held horizontally from front

to back, the other horizontally from right to left, and both are called *a* axes, since they are interchangeable. Three crystal axes intersecting at right angles are, then, used in the tetragonal system, as in the isometric, and they are held in the same positions as in the isometric system. The two horizontal axes are interchangeable, but, unlike the conditions in the isometric system, neither is interchangeable with the vertical axis (see Fig. 56).

Orienting Crystals.

Holohedral tetragonal forms are oriented by holding the principal symmetry plane horizontally, and either set of interchangeable secondary symmetry planes vertically from front to back and from right to left. The crystal axes will then extend in the proper directions.

First Order Position Defined.

Forms with faces or faces extended that cut the two horizontal crystal axes equally (at equal finite distances from the origin) are said to be in the first order position.

Second Order Position Defined.

Forms with faces or faces extended parallel to one (and only one) horizontal axis are said to be in the second order position.

Third Order Position Defined.

Forms with faces or faces extended that cut the two horizontal axes unequally at finite distances are said to be in the third order position.

Holohedral Tetragonal Forms Tabulated.

Name.	Symbol.	Number of faces.
1st order pyramid (Fig. 57)	$a:a:mc$	8
1st order prism (Fig. 58)	$a:a:\infty c$	4
2nd order pyramid (Fig. 59)	$a:\infty a:mc$	8
2nd order prism (Fig. 60)	$a:\infty a:\infty c$	4
Ditetragonal pyramid (Fig. 61)	$a:na:mc$	16
Ditetragonal prism (Fig. 62)	$a:na:\infty c$	8
Basal-pinacoid (Fig. 63)	$\infty a:\infty a:c$	2

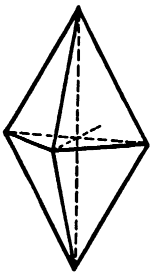


FIG. 57. — 1st order pyramid.

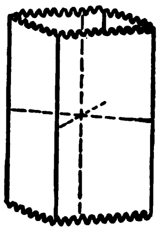


FIG. 58. — 1st order prism.

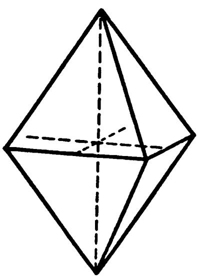


FIG. 59. — 2nd order pyramid.

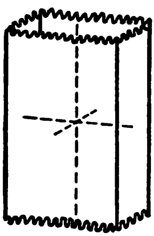


FIG. 60. — 2nd order prism.

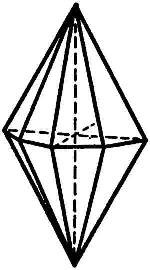


FIG. 61. — Ditetragonal pyramid.

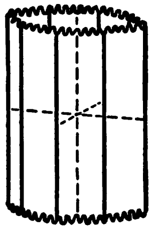


FIG. 62. — Ditetragonal prism.

Synonyms for the Names of the Holohedral Tetragonal Forms.

1st order pyramid — direct pyramid, 1st order bipyramid, or unit pyramid.

1st order prism — direct prism or unit prism.

2nd order pyramid — indirect pyramid or diametral pyramid.

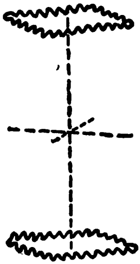


FIG. 63. — Basal pinacoid.

2nd order prism — indirect prism or diametral prism.

Ditetragonal pyramid — ditetragonal bipyramid or zirconoid.

Ditetragonal prism — none.

Basal-pinacoid — basal-plane.

Method of Determining Holohedral Tetragonal Forms by the Use of Symbols.

After properly orienting the crystal in the manner already described select any face in the upper right

octant facing the observer, and ascertain the relative distances at which its plane intersects the three crystal axes, remembering that no face or face extended can cut the vertical axis at the same distance from the origin as it cuts either horizontal axis. If, for instance, it appears that the plane of the face selected intersects the three axes, but that the two horizontal axes are not cut at the same distance from the origin, the symbol of that face (and of the form of which it is a part) is $a : na : mc$. By referring to the table of holohedral tetragonal forms (which should be memorized as soon as possible) it is seen that the form is the ditetragonal pyramid. If more than one form is represented on the crystal, each may be determined in the same way.

Suggestions for Attaining Facility in the Recognition of Forms.

The holohedral tetragonal forms are so easily identified after they have been properly oriented that it seems almost unnecessary to offer rules aiming toward their rapid recognition.

The following statements may, however, prove useful to the beginner.

1st order pyramid: A single face lying wholly within an octant.

1st order prism: A vertical face cutting both horizontal axes equally.

2nd order pyramid: A face sloping down from the vertical axis directly towards the observer.

The 2nd order pyramid differs in no way from the 1st order pyramid excepting in position with respect to the horizontal crystal axes; and an 8-

faced pyramid may be placed in either the 1st or 2nd order position at will. Such a pyramid may, then, be considered either a 1st or 2nd order pyramid depending upon the set of interchangeable symmetry axes with which the crystal axes are chosen to coincide. It is only when forms in both the 1st and 2nd order positions are present on a crystal that it is necessary to distinguish between 1st and 2nd order pyramids.

2nd order prism: A vertical face extending from right to left or front to back.

As is the case with the 2nd order pyramid, a 2nd order prism differs in no way from a 1st order prism excepting in position with respect to the horizontal crystal axes; and all that was said in the preceding section relative to the 2nd order pyramid applies with equal truth to the 2nd order prism.

It is customary to select the horizontal crystal axes in such a way as will place the largest and most prominent 8-faced pyramid or 4-faced prism in the 1st order position.

Pyramids and prisms intersecting in horizontal edges are always of the same order.

Ditetragonal pyramid: Two identical faces lying wholly within an octant.

Ditetragonal prism: A vertical face cutting the two horizontal axes unequally.

Basal-pinacoid: A horizontal face on top of the crystal.

Fixed and Variable Forms.

The only fixed holohedral tetragonal forms are the first and second order prism and the basal-pinacoid.

Fixed Angles of the Tetragonal System.

The only fixed angles in this system are those between the fixed forms just mentioned, namely, 90° and 45° (or 135°).

Miscellaneous.

The general statements made in the discussion of the holohedral division of the isometric system regarding combination of forms, determination of the number of forms, repetition of forms on a crystal, and limiting forms applies with equal truth to all the divisions of the tetragonal system. It may be mentioned, however, that repetitions of the same variable form are very common in the tetragonal system, and crystal models showing such repeated forms are not difficult to obtain.

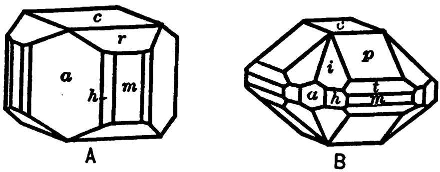


FIG. 64. — Holohedral tetragonal crystals.

A: Basal-pinacoid (*c*), 1st order pyramid (*r*), 1st order prism (*m*), 2nd order prism (*a*), and ditetragonal prism (*h*).

B: Basal-pinacoid (*c*), two 1st order pyramids (*p* and *t*), 1st order prism (*m*), 2nd order prism (*a*), ditetragonal pyramid (*i*), and ditetragonal prism (*h*).

SPHENOIDAL HEMIHEDRAL DIVISION

Development or Derivation of the Forms.

Sphenoidal hemihedral forms may be conceived to be developed by dividing each of the holohedral forms by means of the principal symmetry plane and the *set of secondary symmetry planes containing the crystal axes* into eight parts, or octants, then suppressing all faces lying wholly within alternate parts thus obtained, and extending the remaining faces until they meet in edges or corners.

Symmetry.

Sphenoidal hemihedral forms possess only two interchangeable secondary symmetry planes at right angles to each other.

Selection, Position, and Designation of the Crystal Axes.

The three directions used as crystal axes in the holohedral division are still utilized for the same purpose in the sphenoidal hemihedral division. In other words, the vertical or *c* axis lies at the intersection of the two secondary symmetry planes; while the two interchangeable horizontal or *a* axes, one of which extends from front to back, and the other from right to left, make an angle of 90° with each other, and bisect the angles between the two secondary symmetry planes.

Orienting Crystals.

Sphenoidal hemihedral crystals are oriented in exactly the same way as are tetrahedral hemihedral isometric ones (see p. 30).

Sphenoidal Hemihedral Tetragonal Forms Tabulated.

Name.	Symbol.	Number of faces.	Form from which derived.
±Tetragonal sphenoid (Fig. 65)	$\pm\frac{a : a : mc}{2}$	4	1st order pyramid
±Tetragonal scalenohedron (Fig. 66)	$\pm\frac{a : na : mc}{2}$	8	ditetragonal pyramid
1st order prism (Fig. 58)	$\frac{a : a : \infty c}{2}$	4	1st order prism
2nd order pyramid (Fig. 59)	$\frac{a : \infty a : mc}{2}$	8	2nd order pyramid
2nd order prism (Fig. 60)	$\frac{a : \infty a : \infty c}{2}$	4	2nd order prism
Ditetragonal prism (Fig. 62)	$\frac{a : na : \infty c}{2}$	8	ditetragonal prism
Basal-pinacoid (Fig. 63)	$\frac{\infty a : \infty a : c}{2}$	2	basal-pinacoid

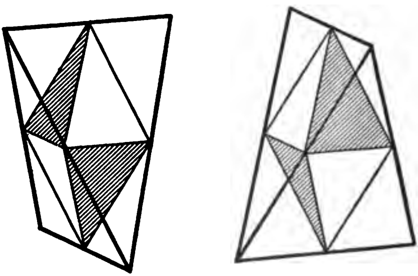


Fig. 65. — Positive (on left) and negative (on right) tetragonal sphenoids containing the forms from which they are derived. The suppressed faces are shaded.

Synonyms for the Names of the Sphenoidal Hemihedral Tetragonal Forms.

Tetragonal sphenoid — hemi-unit pyramid.
Tetragonal scalenohedron — none.

Positive and Negative Forms in the Sphenoidal Hemihedral Tetragonal Division.

+ and − forms are recognized in this division, and are distinguished in exactly the same way as are the + and − forms in the tetrahedral hemihedral

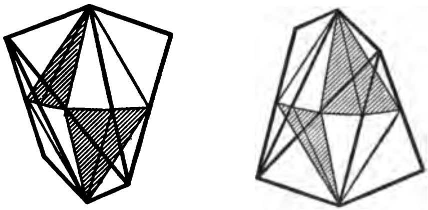


FIG. 66. — Positive (on left) and negative (on right) tetragonal scalenohedrons containing the forms from which they are derived. The suppressed faces are shaded.

division of the isometric system (see p. 32). All that was said there relative to such forms will apply with equal truth to the + and − forms in the division under consideration.

Methods of Determining Sphenoidal Hemihedral Forms by the Use of Symbols.

After properly orienting the crystal in the manner already described all the forms but the tetragonal sphenoid and scalenohedron may be identified easily by applying the rules already given for the determination of holohedral forms of the same name.

The tetragonal sphenoid and scalenohedron may be recognized by determining the symbol of any face in the manner described in the discussion of holohedral forms, dividing this symbol by 2, and then ascertaining from the table the name of the form possessing this symbol.

Suggestions for Attaining Facility in the Recognition of Forms.

Orient the crystal and determine which of the following descriptions (which should be learned at once) apply to the face or faces of different shape or size seen. Call the forms + or − according to the rules already set forth. It is assumed that the student is already familiar with the rules for recognizing those forms identical in shape with the holohedral ones (see p. 90).

Tetragonal sphenoid: A single face in an octant (although not necessarily wholly included therein) sloping down from the vertical axis in such a way as to cut both horizontal axes equally.

Tetragonal scalenohedron: Two faces lying within an octant (although not necessarily wholly included therein) which cut all three axes unequally.

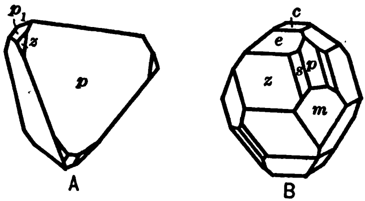


FIG. 67. — Sphenoidal hemihedral tetragonal crystals.

A: + and − tetragonal sphenoids (p and p_1) and 2nd order pyramid (z).

B: Basal pinacoid (c), two 2nd order pyramids (e and z), 1st order prism (m), + tetragonal sphenoid (p), and tetragonal scalenohedron (s).

PYRAMIDAL HEMIHEDRAL DIVISION

Development or Derivation of the Forms.

Pyramidal hemihedral forms may be conceived to be developed by dividing each of the holohedral forms by means of all four secondary symmetry planes into eight parts, then suppressing all faces lying wholly within alternate parts thus obtained, and extending the remaining faces until they meet in edges or corners.

Symmetry.

Pyramidal hemihedral forms possess only one symmetry plane which is in the position of the principal symmetry plane existing in the holohedral division. It is, however, in the pyramidal hemihedral division a secondary rather than a principal symmetry plane since there are no interchangeable symmetry planes perpendicular to it.

Pyramidal hemihedral tetragonal forms may, then, be said to be characterized by the presence of one secondary symmetry plane and a general four-fold arrangement of the faces.

Selection, Position, and Designation of the Crystal Axes.

The vertical or *c* axis is made to coincide with the secondary symmetry axis. Two interchangeable horizontal axes parallel to prominent crystallographic directions at right angles to each other are also selected, one of which is so placed as to extend from front to back, and the other from right to left. Being interchangeable, both are called *a* axes.

Orienting Crystals.

The secondary symmetry plane is held horizontally. The crystal is then rotated around the symmetry axis until the most prominent pyramid or prism lies in the first order position. The crystal axes will then extend in the proper directions.

Pyramidal Hemihedral Tetragonal Forms Tabulated.

Name.	Symbol.	Number of faces.	Form from which derived.
3rd order pyramid (Fig. 68)...	$\frac{a : na : mc}{2}$	8	ditetragonal pyramid
3rd order prism (Fig. 69).....	$\frac{a : na : \infty c}{2}$	4	ditetragonal prism
1st order pyramid (Fig. 57)...	$\frac{a : a : mc}{2}$	8	1st order pyramid
1st order prism (Fig. 58)......	$\frac{a : a : \infty c}{2}$	4	1st order prism
2nd order pyramid (Fig. 59)..	$\frac{a : \infty a : mc}{2}$	8	2nd order pyramid
2nd order prism (Fig. 60).....	$\frac{a : \infty a : \infty c}{2}$	4	2nd order prism
Basal-pinacoid (Fig. 63).......	$\frac{\infty a : \infty a : c}{2}$	2	basal-pinacoid

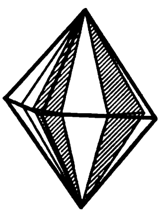


FIG. 68. — 3rd order pyramid containing the form from which it is derived. Suppressed faces are shaded.

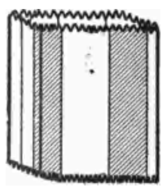


FIG. 69. — 3rd order prism containing the form from which it is derived. Suppressed faces are shaded.

Synonyms for the Names of the Pyramidal Hemihedral Tetragonal Forms.

3rd order pyramid — square pyramid of the third order or third order bipyramid.

3rd order prism — square prism of the third order.

Method of Determining Pyramidal Hemihedral Tetragonal Forms by the Use of Symbols.

After properly orienting the crystal in the manner already described all the forms but the 3rd order pyramid or prism may be identified easily by applying the rules already given for the determination of holohedral forms of the same name. The 3rd order pyramid or prism may be recognized by determining the symbol of any face in the manner already described in the discussion of holohedral isometric forms, dividing this symbol by 2, and then ascertaining from the table the name of the form possessing this symbol.

Suggestions for Attaining Facility in the Recognition of Forms.

Orient the crystal and determine which of the following descriptions (which should be learned at once) apply to the face or faces of different shape or size seen. It is possible, but unnecessary, to distinguish between positive and negative forms in this division. It is assumed that the student is already familiar with the rules for recognizing those forms identical in shape and position with the holohedral ones (see p. 90).

3rd order pyramid: A single face lying in an octant (although not necessarily wholly included therein)

which intersects all three crystal axes at different finite distances from the origin.

The 3rd order pyramid differs in no way from the 1st or 2nd order pyramid excepting in position with respect to the horizontal crystal axes. All three types of 8-faced pyramids may have the same appearance; and any such pyramid may be held at will as a 1st, 2nd, or 3rd order pyramid. The 3rd order pyramid is skewed or twisted through a small

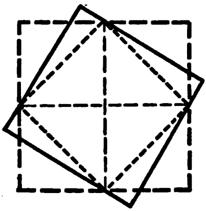


FIG. 70. — Diagram showing the relations of the 1st order (dotted lines), 2nd order (broken lines), and 3rd order (solid lines) pyramids and prisms to the horizontal crystal axes.

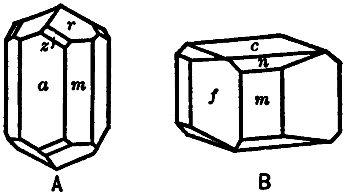


FIG. 71. — Pyramidal hemihedral hexagonal crystals.

A: 1st order pyramid (*r*), 1st order prism (*m*), 2nd order prism (*a*), and 3rd order pyramid (*z*).

B: Basal-pinacoid (*c*), 1st order pyramid (*n*), 1st order prism (*m*), and 3rd order prism (*f*).

angle (less than 45°) either to the right or left away from the position of the 1st or 2nd order pyramid. Fig. 70 shows how the horizontal axes are cut by 1st, 2nd, and 3rd order pyramids and prisms.

3rd order prism: A single face parallel to the vertical axis, which intersects the two horizontal axes at unequal distances from the origin.

All that was said in the preceding section relative to the 3rd order pyramid applies with equal truth to the 3rd order prism.

Application of the Law Governing Combination of Forms.

It has already been noted (see p. 41) that one of the commonest mistakes made in determining crystal forms is the naming of two or more forms which cannot possibly occur on the same crystal, such, for instance, as the sphenoid and 3rd order pyramid. The presence on the same crystal of two forms like these, which are peculiar to different divisions of the system, is, of course, in direct violation of the law governing the combination of forms (see p. 36). A student should thoroughly familiarize himself with the following table if he wishes to avoid the mistake mentioned.

Holohedral forms.	Corresponding sphenoidal hemihedral forms.	Corresponding pyramidal hemihedral forms.
1st order pyramid........	*tetragonal sphenoid*	1st order pyramid
1st order prism...........	1st order prism	1st order prism
2nd order pyramid.......	2nd order pyramid	2nd order pyramid
2nd order prism..........	2nd order prism	2nd order prism
Ditetragonal pyramid....	*tetragonal scalenohedron*	*3rd order pyramid*
Ditetragonal prism.......	ditetragonal prism	*3rd order prism*
Basal-pinacoid...........	basal-pinacoid	basal-pinacoid

It will be noted from the above table that the 1st order prism, 2nd order pyramid and prism, and the basal pinacoid occur in all three of the divisions already discussed, and may, therefore, be combined with any other forms in these divisions. Further, it will be seen that the 1st order pyramid and ditetragonal prism occur unchanged in name or shape in two of the divisions; while the ditetragonal pyramid occurs only as a holohedral form.

TRAPEZOHEDRAL HEMIHEDRAL DIVISION

Trapezohedral hemihedral forms may be conceived to be developed by dividing each holohedral form by the principal and all the secondary symmetry planes into sixteen parts, then suppressing all faces lying wholly within alternate parts thus obtained, and extending the remaining faces until they meet in edges or corners.

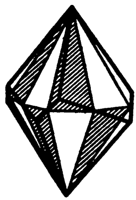


FIG. 72.—Tetragonal trapezohedron containing the form from which it is derived. The suppressed faces are shaded.

As the ditetragonal pyramid is the only tetragonal form with sixteen faces, it is evident that a ditetragonal pyramid face is the only one that can lie wholly within one of the parts obtained by dividing a tetragonal crystal in the manner just specified. The ditetragonal pyramid is, then, the only tetragonal form from which a trapezohedral hemihedral form differing from the holohedral one in shape and in name can be derived. This new form is called the tetragonal trapezohedron (Fig. 72). Since no mineral is known to crystallize in this division, its further consideration seems unnecessary.

Table of Holohedral Tetragonal Symbols Used by Various Authorities.

	Weiss.	Naumann.	Dana.	Miller.
1st order pyramid.........	$a : a : mc$	mP	m	hhl
1st order prism............	$a : a : \infty c$	∞ P	I	110
2nd order pyramid........	$a : \infty a : mc$	mP ∞	m-i	$h0l$
2nd order prism...........	$a : \infty a : \infty c$	∞ P ∞	i-i	100
Ditetragonal pyramid.....	$a : na : mc$	mPn	m-n	hkl
Ditetragonal prism........	$a : na : \infty c$	∞ Pn	i-n	$hk0$
Basal-pinacoid.............	$\infty a : \infty a : c$	0P	O	001

For methods of forming hemihedral symbols, see page 44.

CHAPTER V

ORTHORHOMBIC SYSTEM

HOLOHEDRAL DIVISION

Symmetry.

The holohedral division of the orthorhombic system is characterized by the presence of three non-interchangeable secondary symmetry planes at right angles to each other.

The Selection, Position, and Designation of the Crystal Axes.

The three crystal axes utilized in this system are so chosen as to coincide with the secondary symmetry

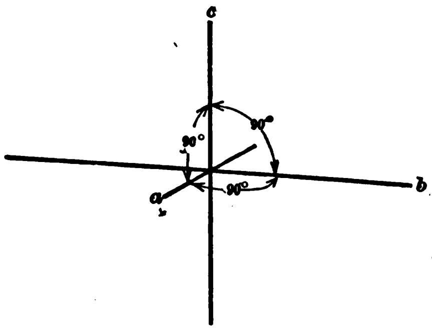


FIG. 73. — Crystal axes of the orthorhombic system.

axes. One is held vertically and is called the vertical or *c* axis; another is held horizontally from right to left, and is called the macro (long) or *b* axis; while

the third extends horizontally from front to back, and is called the brachy (short) or *a* axis. None of the three crystal axes are interchangeable (see Fig. 73).

Orienting Crystals.

One symmetry plane is held so as to extend vertically from front to back, another vertically from right to left, and the third horizontally. The crystal axes will then extend in the proper directions, and the forms can be named according to the directions given later. There are, however, certain conventions that are set forth in immediately succeeding paragraphs, and which should be observed as closely as possible. In considering the statements that follow, it should be remembered that the relative lengths of the axes are determined by the distances from the origin to the points where the plane of a face of the ground-form intersects each crystal axis.

If a crystal is decidedly elongated, the longest axis becomes the *c* axis; while, if it is notably tabular, the shortest axis is used as the *c* axis. When neither elongated nor tabular an axis of intermediate length is used for the *c* axis.

The *c* axis having been selected, the longer of the other two axes is held from right to left as the macro or *b* axis, and the shorter from front to back as the brachy or *a* axis.

Since the student is unable to determine which form is the ground-form, and since the ground-form may not be represented on some crystals, it is permissible to determine the relative lengths of the

axes by noting the distances from the origin at which prominent faces intersect two or three axes.

Holohedral Orthorhombic Forms Tabulated.

Name.	Symbol.	Number of faces.
Pyramid (Fig. 74)	$na : b : mc$	8
Prism (Fig. 75)	$na : b : \infty c$	4
Macro-dome (Fig. 76)	$a : \infty b : mc$	4
Brachy-dome (Fig. 77)	$\infty a : b : mc$	4
Macro-pinacoid (Fig. 78)	$a : \infty b : \infty c$	2
Brachy-pinacoid (Fig. 79)	$\infty a : b : \infty c$	2
Basal-pinacoid (Fig. 80)	$\infty a : \infty b : c$	2

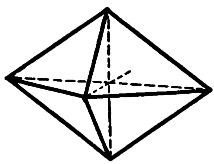


Fig. 74. — Pyramid.

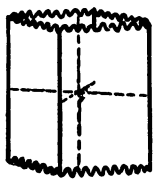


Fig. 75. — Prism.

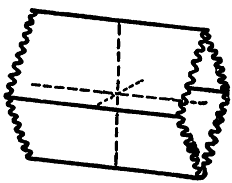


Fig. 76. — Macro-dome.

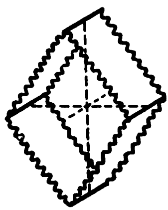


Fig. 77. — Brachy-dome.

Remarks on the Holohedral Orthorhombic Forms.

It will be seen by examining the table just given that the holohedral orthorhombic forms may be grouped into three divisions, one containing the pyramid with eight faces cutting all three axes at

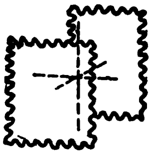


FIG. 78. — Macro-pinacoid.

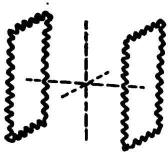


FIG. 79. — Brachy-pinacoid.

finite distances, one containing the prism and domes with four faces parallel to one axis and cutting the other two at finite distances, and one containing the pinacoids with two faces parallel to two axes. The forms in any one of these divisions may be changed into any other form in the same division by a different selection of crystal axes; but a form in no one division can be named as a form in any of the other divisions no matter how the crystal is held. The pyramid is the only holohedral orthorhombic form which will when unmodified completely bound the crystal. Since all the forms but the pinacoids (the fixed forms) contain one or more variables in their symbols, they may be repeated an indefinite number of times on the same crystal.

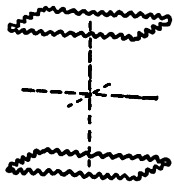


FIG. 80. — Basal-pinacoid.

Domes and pinacoids (excepting the basal-pinacoid) are named by prefixing the name of the horizontal axis to which they are parallel.

Synonyms for the Names of the Holohedral Orthorhombic Forms.

Pyramid — unit-pyramid, macro-pyramid, and brachy-pyramid.
Prism — unit-prism, macro-prism, and brachy-prism.
Macro-dome — none.
Brachy-dome — none.
Macro-pinacoid — none.
Brachy-pinacoid — none.
Basal-pinacoid — basal-plane.

Method of Determining Holohedral Orthorhombic Forms by the Use of Symbols.

The usual method of determining forms by the use of symbols, as presented in the discussion of the systems already described, may be used successfully in the orthorhombic system. In forming symbols it should be remembered that the distance from the origin to the point where the plane of any face intersects the macro or b axis is always made b (or ∞b if the face is parallel to this axis); while the distance from the origin to the point where the plane of the face intersects the brachy or a axis is called na (or ∞a) if the plane cuts the b axis at a finite distance from the origin, and is called a (or ∞a) if the plane cuts the b axis at infinity. Similarly, the distance from the origin to the point where the plane of a face intersects the c axis is called mc (or ∞c) if the plane cuts either the a or b axes at

finite distances; while it is called *c* if both the *a* and *b* axes are cut at infinity.

Suggestions for Attaining Facility in the Recognition of Forms.

Orient the crystal and determine which of the following descriptions (which should be learned at once) apply to the face or faces of different shape or size seen.

Pyramid: A face lying entirely within an octant.

Prism: A vertical face oblique to both horizontal axes.

Macro-dome: A face sloping from the vertical axis directly down toward the observer.

Brachy-dome: A face sloping from the vertical axis down to the right or left.

Macro-pinacoid: A vertical face extending from right to left.

Brachy-pinacoid: A vertical face extending from front to back.

Basal-pinacoid: A horizontal face on top of the crystal.

Fixed and Variable Forms.

The only fixed holohedral orthorhombic forms are the three pinacoids.

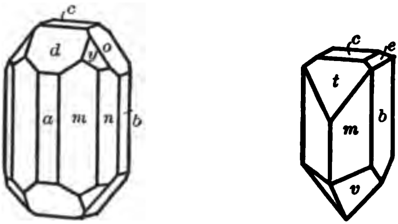
Fixed Angles of the Orthorhombic System.

The only fixed angle in this system is that between the three pinacoids, namely, 90°.

Miscellaneous.

The general statements made in the discussion of the holohedral division of the isometric system

regarding combination of forms, determination of the number of forms, and limiting forms apply with equal truth to all the divisions of the tetragonal system. Hemimorphism (see p. 17) is shown by crystals of calamine as well as by those of certain rare minerals.



A B

FIG. 81. — Holohedral orthorhombic crystals.

A: Basal-pinacoid (*c*), macro-dome (*d*), macro-pinacoid (*a*), brachy-dome (*o*), brachy-pinacoid (*b*), pyramid (*y*), and two prisms (*m* and *n*).

B (hemimorphic): Basal-pinacoid (*c*), brachy-dome (*e*), brachy-pinacoid (*b*), macro-dome (*t*), and prism (*m*). On other end: pyramid (*v*).

SPHENOIDAL HEMIHEDRAL DIVISION

Development or Derivation of the Forms.

Sphenoidal hemihedral forms may be conceived to be developed by dividing each of the holohedral forms by means of the three secondary symmetry planes into eight parts, then suppressing all faces lying wholly within alternate parts thus obtained, and extending the remaining faces until they meet in edges or corners.

Symmetry.

Sphenoidal hemihedral forms possess no symmetry planes whatever, but are characterized by the presence of three

prominent, non-interchangeable crystallographic directions at right angles to each other.

Selection, Position, and Designation of the Crystal Axes.

Three prominent, non-interchangeable crystallographic directions at right angles to each other are selected as the crystal axes. These are held and named exactly as in the holohedral division.

Orienting Crystals.

Usually the easiest way to orient sphenoidal hemihedral crystals is to identify by general appearance some form whose holohedral and hemihedral shapes are the same, and to hold this form in the position it occupies in the holohedral division. It is often just as easy or easier to find three prominent, non-interchangeable crystallographic directions at right angles to each other, and to hold these in the positions of the crystal axes, in the manner already set forth in the discussion of the holohedral division.

Sphenoidal Hemihedral Orthorhombic Forms Tabulated.

Name.	Symbol.	Number of faces.	Form from which derived.
±Orthorhombic sphenoid (Fig. 82)	$\pm\frac{na:b:mc}{2}$	4	pyramid
Prism (Fig. 75)	$\frac{na:b:\infty c}{2}$	4	prism
Macro-dome (Fig. 76)	$\frac{a:\infty b:mc}{2}$	4	macro-dome
Brachy-dome (Fig. 77)	$\frac{\infty a:b:mc}{2}$	4	brachy-dome
Macro-pinacoid (Fig. 78)	$\frac{a:\infty b:\infty c}{2}$	2	macro-pinacoid
Brachy-pinacoid (Fig. 79)	$\frac{\infty a:b:\infty c}{2}$	2	brachy-pinacoid
Basal-pinacoid (Fig. 80)	$\frac{\infty a:\infty b:c}{2}$	2	basal-pinacoid

Synonyms for the Names of Sphenoidal Hemihedral Tetragonal Forms.

Orthorhombic sphenoid — none.

Positive and Negative Forms in the Sphenoidal Hemihedral Division.

+ and − forms are recognized in this division, and are distinguished in exactly the same way as are the + and − forms in the tetrahedral hemihedral division of the isometric system (see p. 32). All that was said there relative to such forms will apply with equal truth to the + and − forms in the division under consideration.

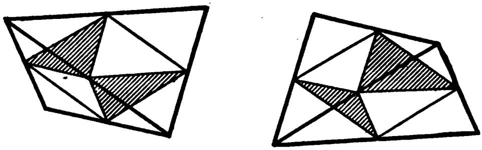


FIG. 82. — Positive (on left) and negative (on right) orthorhombic sphenoids containing the forms from which they are derived. The suppressed faces are shaded.

Method of Determining Sphenoidal Hemihedral Forms by the Use of Symbols.

After properly orienting the crystal in the manner already described all the forms but the orthorhombic sphenoid may be identified easily by applying the rules already given for the determination of holohedral forms of the same name.

The orthorhombic sphenoid may be recognized by determining the symbol of any face in the manner described in the discussion of holohedral forms, dividing this symbol by two, and then ascertaining from the table the name of the form possessing this symbol.

Suggestions for Attaining Facility in the Recognition of Forms.

After properly orienting the crystal all the forms but the orthorhombic sphenoid may be recognized by the rules al-

ready given for identifying the holohedral forms (see p. 108). The following description of the position of an orthorhombic sphenoid face should be learned at once; and if faces answering to this description are present on a crystal, they should be called + or – according to the rules already set forth.

Orthorhombic Sphenoid: A single face in an octant (although not necessarily wholly included therein) sloping down from the vertical axis so as to cut both horizontal axes obliquely.

Table of Holohedral Orthorhombic Symbols Used by Various Authorities.

	Weiss.	Naumann.	Dana.	Miller.
Pyramid	$na : b : mc$	m or mPn	1 or $m\text{-}n$	111 or hkl
Prism	$na : b : \infty c$	mP or ∞Pn	I or $i\text{-}n$	110 or $hk0$
Macro-dome	$a : \infty b : mc$	$mP\bar{\infty}$	$m\text{-}\bar{i}$	$h0l$
Brachy-dome	$\infty a : b : mc$	$mP\breve{\infty}$	$m\text{-}\breve{i}$	$0kl$
Macro-pinacoid	$a : \infty b : \infty c$	$\infty P\bar{\infty}$	$i\text{-}\bar{i}$	100
Brachy-pinacoid	$\infty a : b : \infty c$	$\infty P\breve{\infty}$	$i\text{-}\breve{i}$	010
Basal-pinacoid	$\infty a : \infty b : c$	0P	O	001

For methods of forming hemihedral symbols, see p. 44.

CHAPTER VI

MONOCLINIC SYSTEM

HOLOHEDRAL DIVISION

Symmetry.

The holohedral division of the monoclinic system is characterized by the presence of one secondary symmetry plane.

Selection, Position, and Designation of the Crystal Axes.

The ortho or *b* crystal axis is made to coincide with the secondary symmetry axis, and is held horizontally from right to left. The other two axes (which are made to pass through the geometric center of the crystal) are selected lying in the symmetry plane parallel to two prominent crystallographic directions as nearly at right angles to each other as possible. One is held vertically and called the vertical or *c* axis; while the other is held so as to slope or incline *down* toward the observer, and is called the clino or *a* axis (see Fig. 83).

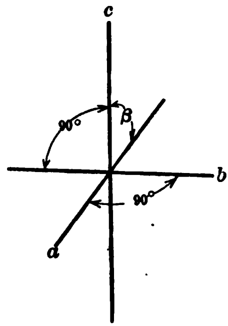


FIG. 83. — Crystal axes of the monoclinic system. β is a variable angle, but can never be equal to a fixed angle of any system.

None of the crystal axes are interchangeable.

The ortho axis makes right angles with both the

vertical and the clino axes, but the clino and vertical axes are never exactly at right angles to each other.

Orienting Crystals.

The symmetry plane is held so as to extend vertically from front to back. The crystal is then rotated around the symmetry (ortho) axis until the most prominent crystallographic direction (which fixes the position of the *c* axis) is held vertically, and a second prominent crystallographic direction (which fixes the position of the clino axis) is held so as to slope or incline down toward the observer.

Prominent crystallographic directions may be edges, the intersection of the symmetry plane and faces, or lines connecting opposite corners or the middle points of opposite edges or faces. It will usually be found desirable to select the vertical and clino axes parallel to prominent *edges*, unless by so doing the two axes mentioned are forced to intersect in a decidedly acute angle.

Holohedral Monoclinic Forms Tabulated.

Name.	Symbol.	Number of faces.
+Pyramid (Fig. 84)	$-na : b : mc$	4
−Pyramid (Fig. 85)	$+na : b : mc$	4
Prism (Fig. 86)	$na : b : \infty c$	4
Clino-dome (Fig. 87)	$\infty a : b : mc$	4
Ortho-pinacoid (Fig. 88)	$a : \infty b : \infty c$	2
+Ortho-dome (Fig. 89)	$-na : \infty b : mc$	2
−Ortho-dome (Fig. 90)	$+na : \infty b : mc$	2
Basal-pinacoid (Fig. 91)	$\infty a : \infty b : c$	2
Clino-pinacoid (Fig. 92)	$\infty a : b : \infty c$	2

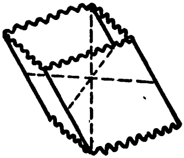


FIG. 84. — Positive pyramid.

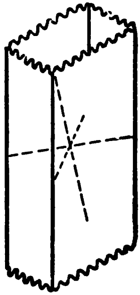


FIG. 85. — Negative pyramid.

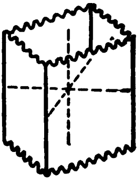


FIG. 86.
Prism.

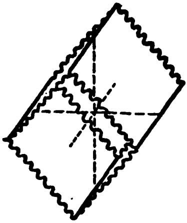


FIG. 87.
Clino-dome.

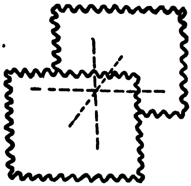


FIG. 88.
Ortho-pinacoid.

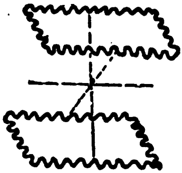


FIG. 89. — Positive ortho-dome.

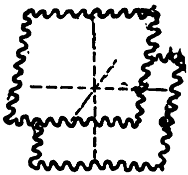


FIG. 90. — Negative ortho-dome.

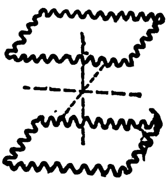


FIG. 91. — Basal-pinacoid.

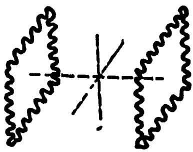


FIG. 92. — Clino-pinacoid.

Remarks on the Holohedral Monoclinic Forms.

It will be seen by examining the table just given that the holohedral monoclinic forms may be grouped into three divisions, one containing forms whose faces intersect the symmetry plane obliquely, namely, the + and − pyramid, clino-dome, and prism; another containing forms whose faces are perpendicular to the symmetry plane, namely, the ortho-pinacoid, + and − ortho-dome, and basal-pinacoid; and a third containing a form whose faces are parallel to the symmetry plane, namely, the clino-pinacoid. The forms in any one of these divisions may be changed into any other form in the same division by a different selection of crystal axes; but a form in no one division can be named as a form in any of the other divisions no matter how the crystal is held.

Since all the forms but the pinacoids (the fixed forms) contain one or more variables in their symbols, they may be repeated an indefinite number of times on the same crystal. No one form in this system will completely bound a crystal. In other words, at least two forms must always be represented on a monoclinic crystal.

Domes and pinacoids (excepting the basal-pinacoid) are named by prefixing the name of the horizontal axis to which they are parallel.

Synonyms for the Names of the Holohedral Monoclinic Forms.

Pyramid — unit-, ortho-, and clino-hemipyramid.
Prism — unit-, ortho-, and clino-prism.
Clino-dome — none.
Ortho-pinacoid — none.
Ortho-dome — none.
Basal-pinacoid — basal-plane.
Clino-pinacoid — none.

Positive and Negative Forms in the Holohedral Monoclinic Division.

The + and − forms in this system are distinguished in quite a different manner from the + and − forms in any of the systems already discussed, and are apt to prove quite confusing to a student until he is thoroughly familiar with the conceptions upon which their distinction is based.

It has been decided to call those pyramids and ortho-domes — whose faces or faces extended intersect the clino axis in *front* of the origin (at $+na$); while those pyramids and ortho-domes whose faces or faces extended intersect the clino axis *behind* the origin (at $-na$) are called +. In other words, the + forms lie over the acute angle (β, Fig. 83) formed by the intersection of the vertical and the clino axes, while the − forms lie over the obtuse angle between these axes.

It should be noted that the + forms have a − sign, while the − forms have a + sign in their symbols. This inconsistency is unfortunate, but the practice of naming these forms in the manner specified has become so firmly established that it

appears impossible to change the nomenclature. The following rule will be found useful in distinguishing between + and − pyramids:

A + pyramid intersects the basal-pinacoid on top of a crystal in edges which converge *away* from the observer.

A − pyramid intersects a basal-pinacoid on top of a crystal in edges which converge *toward* the observer.

Reason Why Faces on Top or in Front of a Crystal are Duplicated at the Bottom or Back.

It is easy to understand why the clino-pinacoid faces are duplicated on both sides of a crystal, since the presence of a symmetry plane requires such duplication. It is not so easy to understand, however, why the faces of the other forms are duplicated on the top and bottom and front and back, since no symmetry plane lies between such duplicated faces. The reason for this duplication is found in the law of axes which states that opposite ends of crystal axes must be cut by the same number of similar faces similarly placed. In order that this law shall hold good no matter how the vertical and clino axes are chosen, it is necessary that faces be duplicated in the manner mentioned.

Method of Determining Holohedral Monoclinic Forms by the Use of Symbols.

The method outlined in the presentation of the orthorhombic system (see p. 107) may be applied with equal facility to the monoclinic system, although the conception of + and − forms already

outlined must be borne in mind when naming pyramids and ortho-domes.

Suggestions for Attaining Facility in the Recognition of Forms.

Orient the crystal and determine which of the following descriptions apply to the face or faces of different shape or size seen.

+ *Pyramid:* A face whose plane cuts all three axes at finite distances from the origin, and the clino axis behind the vertical axis.

− *Pyramid:* A face whose plane cuts all three axes at finite distances from the origin, and the clino axis in front of the vertical axis.

Clino-dome: A face sloping from the vertical axis down to the right or left and parallel to the clino axis.

A clino-dome may often be distinguished with ease from the + or − pyramid if it is remembered that its faces intersect a basal-pinacoid or another clino-dome in edges that are parallel.

Prism: A vertical face oblique to the ortho axis.

Ortho-pinacoid: A vertical face extending from right to left.

+ *Ortho-dome:* A face whose plane is parallel to the ortho axis, and cuts the clino axis behind the origin.

− *Ortho-dome:* A face sloping down from the vertical axis directly toward the observer, and cutting the clino axis in front of the origin.

Basal-pinacoid: A face sloping down from the vertical axis directly toward the observer and parallel to the clino axis.

Clino-pinacoid: A vertical face extending from front to back.

Fixed and Variable Forms.

The only holohedral monoclinic fixed forms are the three pinacoids.

Fixed Angles of the Monoclinic System.

The only fixed angle in this system is that between the clino- and the ortho- or basal-pinacoid, namely, 90°.

Miscellaneous.

The general statements made in the discussion of the holohedral division of the isometric system regarding combination of forms, determination of the number of forms, and limiting forms apply with equal truth to this system.

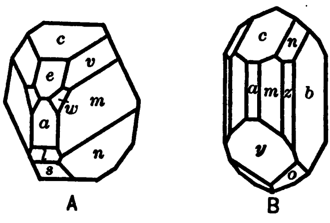


FIG. 93. — Holohedral monoclinic crystals.

A: (clino axis selected parallel to the face lettered *c*): basal-pinacoid (*c*), − ortho-dome (*e*), two + ortho-domes (*l* and *s*), two − pyramids (*v* and *w*), + pyramid (*n*), prism (*m*), and ortho-pinacoid (*a*).

B: (clino axis selected parallel to the face lettered *c*): basal-pinacoid (*c*), + ortho-dome (*y*), ortho-pinacoid (*a*), clino-dome (*n*), clino-pinacoid (*b*), + pyramid (*o*), and two prisms (*m* and *z*).

Hemihedral and Hemimorphic Forms.

Hemihedral and hemimorphic forms are too rare to require consideration.

Table of Holohedral Monoclinic Symbols Used by Various Authorities.

	Weiss.	Naumann.	Dana.	Miller.
+Pyramid.......	$-na : b : mc$	$+mP$ or $+mPn$	$+m$ or $+m\text{-}n$	$\bar{h}hl$ or $\bar{h}kl$
−Pyramid.......	$+na : b : mc$	$-mP$ or $-mPn$	$-m$ or $-m\text{-}n$	hhl or hkl
Prism............	$na : b : \infty c$	∞P or ∞Pn	I or $i\text{-}n$	110 or $hk0$
Clino-dome......	$\infty a : b : mc$	$mP\grave{\infty}$	$m\text{-}\grave{i}$	$0kl$
Ortho-pinacoid...	$a : \infty b : \infty c$	$\infty P\bar{\infty}$	$i\text{-}\bar{i}$	100
+Ortho-dome....	$-na : \infty b : mc$	$+mP\bar{\infty}$	$+m\text{-}\bar{i}$	$\bar{h}0l$
−Ortho-dome....	$+na : \infty b : mc$	$-mP\bar{\infty}$	$-m\text{-}\bar{i}$	$h0l$
Basal-pinacoid....	$\infty a : \infty b : c$	$0P\grave{\infty}$	O	001
Clino-pinacoid....	$\infty a : b : \infty c$	$\infty P\bar{\infty}$	$i\text{-}\grave{i}$	000

CHAPTER VII

TRICLINIC SYSTEM

HOLOHEDRAL DIVISION

Symmetry.

The triclinic system is characterized by the absence of any kind of symmetry plane.

Selection, Position, and Designation of the Crystal Axes.

Three crystal axes are selected parallel to prominent crystallographic directions (see p. 114) and at as nearly right angles to each other as possible. These are held and named exactly as in the orthorhombic system (see p. 103) excepting that the macro axis extending from right to left and the brachy axis extending from front to back will not be horizontal; and none of the axes are at right angles to each other, nor can the angles between them be the fixed angles of any system.

Orienting Crystals.

The three crystal axes having been selected, the one chosen as the *c* axis is held vertically; the shorter of the other two axes (the brachy axis) is so held as to slope down directly toward the observer. The macro axis will, then, extend from right to left, intersecting the plane through the vertical and brachy axes more or less obliquely.

The *c* axis is selected according to the conventions already given in the discussion of the orthorhombic system (see p. 104).

Triclinic Forms Discussed.

The triclinic forms have exactly the same names and symbols as the corresponding orthorhombic forms (see p. 105), but differ therefrom in that each triclinic form consists of only a single pair of parallel and opposite faces.

Since each form in this system consists of but two faces, it follows that the forms differ from one another only as regards their position with respect to the crystal axes; and any form may be changed into any other form by selecting the crystal axes so as to run in the proper direction. If, however, the conventions with respect to the choice of the axes are observed, different observers will in most cases designate all the forms by the same names.

Synonyms for the Names of the Triclinic Forms.

Pyramid — unit-, brachy-, and macro-tetrapyramid.
Prism — unit-, brachy-, and macro-hemiprism.
Macro-dome — hemi-macro-dome.
Brachy-dome — hemi-brachy-dome.
Macro-pinacoid — none.
Brachy-pinacoid — none.
Basal-pinacoid — basal-plane.

Reason Why All Triclinic Forms Consist of *Two* Parallel Faces.

It is not at first easy to see why a face on one side of a triclinic crystal must be duplicated on the

opposite side, since no symmetry plane lies between these faces; but the law of axes states that opposite ends of crystal axes must be cut by the same number of similar faces similarly placed; and, in order that this law shall hold good no matter how the crystal axes are chosen, it is necessary that faces be duplicated in the manner mentioned.

Method of Determining Triclinic Forms by the Use of Symbols.

The method already outlined in the discussion of the other crystal systems may be applied with equal facility to the triclinic system, in which it is not necessary to distinguish between + and − forms. Care must be taken to give a name to every pair of opposite and parallel faces.

Suggestions for Attaining Facility in the Recognition of Forms.

When the axes are nearly perpendicular to each other, it is possible to determine the forms by slightly modifying the rules already given for determining those of the same name in the orthorhombic system (see p. 108). The modifications required are those introduced by the fact that none of the axes lie at right angles to each other. In many cases, however, the forms can be determined most readily by noting the relationship of the faces with respect to the axes, which is really equivalent to determining the symbol of each face. The following descriptions of the forms are based on their relationship to the axes, and may be applied after a crystal is properly oriented.

Pyramid: A face whose plane cuts all three crystal axes at finite distances from the origin.

In order to determine the number of pyramids, it is necessary to count all the pyramidal faces lying above a plane passed through the macro and brachy axes.

Prism: A vertical face whose plane intersects both the macro and brachy axes at finite distances from the origin.

In order to determine the number of prisms, it is necessary to count all the prismatic faces lying in front of a plane passed through the macro and vertical axes.

Macro-dome: A face whose plane is parallel to the macro axis and intersects the vertical and the brachy axes at finite distances from the origin.

In order to determine the number of macro-domes, it is necessary to count all such faces lying above a plane passed through the macro and brachy axes.

Brachy-dome: A face whose plane is parallel to the brachy axis and intersects the macro and vertical axes at finite distances from the origin.

In order to determine the number of brachy-domes, it is necessary to count the number of such faces lying above a plane passed through the macro and brachy axes.

Macro-pinacoid: A vertical face parallel to the macro and vertical axes.

There can be but one macro-pinacoid on a crystal.

Brachy-pinacoid: A vertical face extending from front to back (parallel to the brachy and vertical axes).

There can be but one brachy-pinacoid on a crystal.

Basal-pinacoid: A face (not horizontal) parallel to the macro and brachy axes.

There can be but one basal-pinacoid on a crystal.

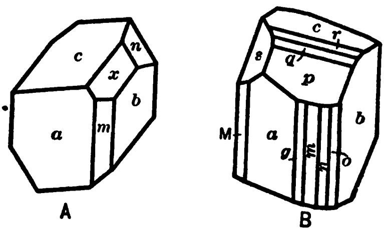


FIG. 94. — Holohedral triclinic crystals.

A: (vertical axis selected parallel to the edge between *a* and *m*, macro axis parallel to the edge between *c* and *a*, and brachy axis parallel to the edge between *b* and *x*): basal-pinacoid (*c*), macro-pinacoid (*a*), brachy-pinacoid (*b*), brachy-dome (*x*), pyramid (*n*), and prism (*m*).

B: (vertical axis selected parallel to the edge between *a* and *M*, macro-axis parallel to the edge between *c* and *a* at the upper right edge of the figure, and brachy axis parallel to the edge between *c* and *b*): basal-pinacoid (*c*), macro-pinacoid (*a*), brachy-pinacoid (*b*), four pyramids (*p*, *q*, *r* and *s*), and five prisms (*M*, *g*, *m*, *n* and *o*).

In order to attain facility in the recognition of triclinic forms, it will be found advisable to practice determining them when the crystal axes are so selected as to make decidedly acute angles with each other. When the student is able to name the forms correctly under such conditions he will find it very easy to do so when the axes are properly selected at as nearly right angles as possible.

Miscellaneous.

The general statements made in the discussion of the holohedral division of the isometric system regarding combination of forms, determination of the number of forms, and limiting forms apply with equal truth to this system.

Hemihedral, Tetartohedral, and Hemimorphic Triclinic Forms.

Since the triclinic system contains no symmetry planes, it is impossible to develop hemihedral or tetartohedral forms according to the general rules already given (see p. 17). Hemimorphic forms are practically unknown.

CHAPTER VIII

TWINS

A Twin Defined.

A twin may be defined as two or more crystals or portions of one crystal so united that, if alternate crystals or portions could be revolved 180° on a so-called twinning plane or planes, one simple untwinned crystal would be formed. See Figs. 95 to 99.

It is, of course, not supposed that Nature actually revolves alternate crystals or portions of a crystal after the simple or untwinned crystals have started to form. The definition just given is, then, merely a statement of tests which may be applied to ascertain whether a given crystal or group of crystals is a twin. Since the cause of the development of twins is unknown, and since there are several classes differing in appearance, it is impossible to formulate a definition based either on genesis or appearance, but some authorities define a twin as two or more crystals or portions of a crystal united according to some definite law.

Most twins are characterized by the presence of re-entrant angles, but the same peculiarity is shown by groups of crystals not united according to the laws of twinning, so this feature cannot be considered distinctive of twins.

Preliminary Definitions.

A Twinning Plane Defined: A twinning plane is a plane so located with reference to two twinned crystals or portions of a crystal that, if one of these crystals or portions of a crystal could be revolved 180° on the plane, the two crystals or portions of a crystal would then be in untwinned relationship to each other (see Fig. 95). A twinning plane is named by stating the name of the possible crystal faces to which it is parallel.

A twinning plane can never be parallel to a symmetry plane excepting in the tetrahedral hemihedral division of the isometric system and the sphenoidal hemihedral division of the tetragonal system, and must be parallel to possible crystal faces.

Twinning Axis Defined: A twinning axis is a line or direction perpendicular to a twinning plane. The twinning axis usually passes through the geometric center of the crystal.

Plane of Union or Composition Face Defined: The plane of union or composition face is a plane along which two crystals or portions of a crystal appear to be united to form a twin. It may or may not coincide in position with the twinning plane. The plane of union must be parallel to a possible crystal form, and is named by stating the name of the possible crystal faces to which it is parallel.

Classes of Twins.

Three classes of twins are generally recognized, namely, contact, interpenetration, and multiple twins. The last named class may be subdivided into subclasses called oscillatory and cyclic twins.

Each of these classes will be discussed in the order mentioned.

Contact Twin Defined: A contact twin is one in which *two portions of a crystal* appear to have been united along a common plane after one portion has been revolved 180° relative to the other (see Fig. 95). The twinning plane and plane of union usually coincide in contact twins.

Contact twins are simpler and commoner than any of the other types, and present no special difficulties.

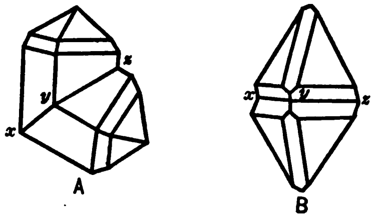


Fig. 95. — Contact twins. A is tetragonal, and B is monoclinic. Positions of the twinning planes indicated by *xyz*.

In studying and reciting upon any type of twin a student should determine and state the following facts:

I. The class of the twin (contact, interpenetration, etc.).

II. The system and division to which the crystal belongs.

III. The forms present on the crystal.

IV. The name of the form whose face or faces the twinning plane parallels.

V. The name of the form whose face or faces the plane of union parallels.

In determining the system of a contact twin and the forms present thereon, it is usually advisable to cover with the hand that portion of the crystal at one side of the twinning plane and to examine only the portion left uncovered. If this is not done, a beginner is apt to be confused by the more or less unsymmetrical arrangement of faces on the two portions of the crystal separated by the twinning plane.

Interpenetration Twin Defined: An interpenetration twin is one in which two or more *complete crystals* appear mutually to penetrate into and through each other according to the laws of twinning (see Fig. 96).

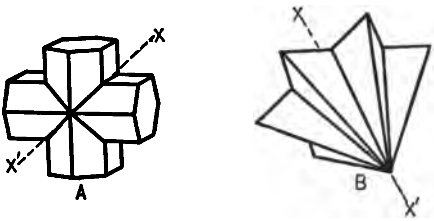


FIG. 96.—Interpenetration twins. A is orthorhombic and B is tetrahedral hemihedral isometric. Positions of the twinning axes indicated by $X-X'$.

It is usually comparatively easy to determine the plane of union and the system and division of such twins, together with the forms present thereon; but it is more difficult to determine the name of the form whose face or faces the twinning plane parallels. It will be found advisable to seek the twinning axis, and, when this is found, determine the name of the form with a face or faces perpendicular to this axis.

Such a form will, of course, have faces parallel to the twinning plane. To determine the position of the twinning axis, hold a pencil with one end against various points on the crystal, and ascertain whether it is possible to bring all points on one of the interpenetrating crystals into the position of identical points on another of the interpenetrating crystals by imagining a rotation of all points on the first crystal 180° around the axis represented by the pencil. If such a rotation would cause the two crystals to coincide, it may be assumed that the pencil is in the position of the twinning axis sought, provided that a plane perpendicular to the pencil is parallel to a possible crystal face.

Multiple Twin Defined: A multiple twin is one in which *more than two portions* of a single crystal appear alternatingly to have been revolved 180° upon parallel or non-parallel twinning planes. Two adjacent parts separated by a twinning plane possess relationships very similar to those of the two parts of a contact twin.

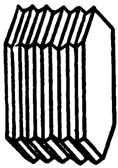


FIG. 97. — Oscillatory multiple twin.

Oscillatory Twin Defined: An oscillatory twin is a multiple twin in which the alternate portions appear to have been revolved 180° upon *parallel* twinning planes (see Fig. 97).

In studying multiple twins, only a portion bounded on one side by a twinning plane and elsewhere by crystal faces should be examined, as other portions included between two parallel twinning planes are apt to possess such a deficiency of faces as to make

the determination of the system, division, and forms difficult or impossible.

Cyclic Twin Defined: A cyclic twin is a multiple twin in which alternate portions appear to have been revolved 180° upon *non-parallel* twinning planes (see Fig. 98).

FIG. 98. — Cyclic multiple twin.

The rotation of alternate parts 180° on non-parallel twinning planes tends to give these twins a ring-like form. In cases where a small number of parts are involved, or where the twinning planes are nearly parallel, the ring will be incomplete; but, when the number of parts are higher or the twinning planes depart considerably from parallelism, a complete ring may result. The center of such a ring of twinned portions may be hollow, or the twinned portions may be in contact in the center. If the cyclic twin forms an incomplete ring, it is comparatively easy to determine all the forms according to the method suggested in the discussion of oscillatory twins; but, if a complete ring is formed, some of the crystal faces necessary in order to cover completely an individual crystal may be missing.

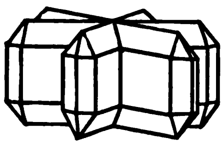


FIG. 99. — Multiple interpenetration twin.

When cyclic twins have the form of complete rings it is customary to give them a name depending upon the number of twinned portions, as trilling, fourling, sixling, eightling, etc.

Interpenetration twins are often of cyclic type as illustrated by Fig. 99.

Influence of Twinning Upon Symmetry.

The twinning plane often appears to be a symmetry plane. Whether this will be the case or not depends upon the character of the crystal and the position of the plane as set forth in the following statement:

The twinning plane will always appear to be a symmetry plane if the crystal is entirely bounded by forms whose faces are arranged in parallel pairs, and if the twinning plane passes through the center of the crystal.

While twinning often apparently introduces a symmetry plane in a crystal, it may also result in the elimination of some symmetry planes. Whether it will have the latter effect or not depends upon the degree of symmetry of the untwinned crystal, and the position of the twinning plane. It may, however, be said that in general twinned crystals belonging to systems or divisions of systems characterized by the presence of more than two symmetry planes often appear to have their symmetry decreased; while crystals belonging to systems characterized by the presence of less than two symmetry planes usually appear to have their symmetry increased. For instance, the twinned holohedral tetragonal crystal shown in Fig. 95A appears to have only two symmetry planes instead of five symmetry planes which the untwinned crystal possesses; while the twinned monoclinic crystal shown in Fig. 95B appears to have two symmetry planes instead of the single one characteristic of the untwinned crystal.

Possible and Impossible Twinning Planes.

It has already been mentioned that a twinning plane must be parallel to a possible crystal face, and cannot be parallel to a symmetry plane. The student will find it a good review of his knowledge of the positions of the symmetry planes and forms in each division of every system if he will attempt to write out a complete list of all the forms parallel to which twinning planes *cannot* lie for the reason that such forms have two or more faces parallel to a symmetry plane or planes. For reference purposes such a list is given below.

Holohedral isometric — cube and dodecahedron.

Tetrahedral hemihedral isometric — no form. (See statement under definition of a twinning plane on p. 129.)

Pentagonal hemihedral isometric — cube.

Holohedral hexagonal — basal-pinacoid, 1st order prism, and 2nd order prism.

Rhombohedral hemihedral hexagonal — 2nd order prism.

Pyramidal hemihedral hexagonal — basal-pinacoid.

Trigonal hemihedral hexagonal — basal-pinacoid and 2nd order prism.

Trapezohedral tetartohedral — no form.

Holohedral tetragonal — basal-pinacoid, 1st order prism, and 2nd order prism.

Sphenoidal hemihedral tetragonal — no form. (See statement under definition of a twinning plane on p. 129.)

Pyramidal hemihedral tetragonal — basal-pinacoid.

Holohedral orthorhombic — macro-, brachy-, and basal-pinacoid.

Sphenoidal hemihedral orthorhombic — no form.

Holohedral monoclinic — clino-pinacoid.

Holohedral triclinic — no form.

CHAPTER IX

MISCELLANEOUS FEATURES

Parallelism of Growth.

When several crystals which may or may not be in contact with each other have all similar faces parallel, parallelism of growth is said to exist. When such parallel-growing crystals are in contact they may mutually interpenetrate, and are then apt to be confused with twins of interpenetration since they bear a superficial resemblance to such twins, and re-entrant angles are common on both.

Cases of parallelism of growth in which the crystals are not in contact are relatively rare, and are difficult to explain. A good illustration of their occurrence is sometimes exhibited by small but well-formed chalcopyrite crystals dotted over the surface of crystallized sphalerite. More frequently a comparatively large crystal appears to be made up of many smaller ones arranged in parallel positions. This is sometimes splendidly shown by large, rather rough, octahedral crystals of fluorite.

Parallelism of Growth and Twinning Differentiated: A group of two or three interpenetrating crystals in parallel position may, as has already been mentioned, be confused with a twin of interpenetration, but may be distinguished therefrom by the fact that it is impossible to find an axis so placed that, if one crystal could be revolved 180° around

it, the rotated crystal would exactly coincide in position with another crystal. As it is not always easy to find the twinning axis, especially in the case of distorted crystals (see p. 144), it will be found easier to base the distinction upon the following facts:

In cases of parallelism of growth, *all* similar faces of all crystals involved will be in parallel positions.

In the case of twins of interpenetration, *some* similar faces of the crystals involved will *not* be in parallel positions although many, perhaps most, such faces may be parallel.

Striations.

Striations Defined: Striations are minute terraces or steps, so small that they often appear like lines etched or drawn upon natural crystal faces.

Groups of such lines in parallel positions are not uncommon on natural crystals, and are often of great service in determining the degree of symmetry of the crystal. They are due to three causes, namely: (1) oscillation of two or more crystal faces; (2) oscillatory twinning; and (3) interference of two crystals in contact with each other. Each will be discussed in the order named. Whenever one face meets another of different slope an edge is formed; and, if two such faces alternate with each other, a series of edges parallel to the first result. Frequent alternation of two such faces is known as oscillation, and produces many parallel edges.

Striations Produced by Oscillation of Faces: Nature sometimes appears to be uncertain as to which

of two faces or forms she prefers to produce, and, instead of one form being more or less prominently modified by the other, many small faces of each form alternate with each other, and form a series of terraces.

As an illustration, consider a mineral (such as pyrite) on crystals of which the cube and pentagonal dodecahedron are equally apt to occur. The cube has a horizontal face on top of the crystal, while the pentagonal dodecahedron has a face sloping gently down toward the observer; and, if the latter striates the former, the cube face will be interrupted by an indefinite number of tiny steps or terraces running from right to left, of which the horizontal strips represent the cube, while the sloping strips represent the pentagonal dodecahedron. If the width of the latter is very small, the cubic shape of the crystal may not be noticeably changed, yet even then the resulting striations may be very distinct, since no light is reflected from the surfaces belonging to the pentagonal dodecahedron when the cube surfaces reflect light. In a similar fashion a pentagonal dodecahedron may be striated by a cube if there is a strong tendency for the former to predominate over the latter.

Almost any face is capable of striating any other face if the forms involved are in the same division of a system, but, in general, it may be said that those forms which intersect at interfacial angles approaching 180° are much more apt to striate each other than are those whose intersections depart considerably from 180°. In fact, faces intersecting at an angle of 135° rarely striate each other, and faces

intersecting at angles of less than 135° almost never do so.

Such striations always conform strictly to the symmetry of the face on which they are found. That is, a group of striations on one side of a symmetry plane must be balanced perfectly by a similar group on the opposite side of such a plane. It follows from the statements just made that striations produced by oscillation of faces may cross a symmetry plane at right angles to it, and may run parallel to a symmetry plane on both sides of it, but *they can never cross a symmetry plane obliquely*. If the striations on one portion of a crystal face are so disposed as to intersect a symmetry plane obliquely, they must be balanced on the opposite side of the symmetry plane by a similar group of striations which meet the first group at an angle, and which are similarly inclined to the symmetry plane. Fig. 100, which shows a cube face striated by a hexoctahedron, illustrates this law.

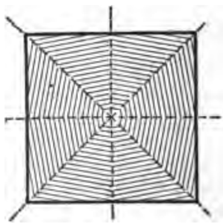


Fig. 100. — Cube face striated by a hexoctahedron. The positions of the symmetry planes are indicated by broken lines.

It is the fact that striations must conform to the symmetry of the crystals on which they occur that makes them of service in determining the symmetry of crystals in cases where the crystal forms present are not distinctive of any particular division of a system. For instance, the cube occurs unchanged in appearance in all divisions of the isometric system, but a cube striated like Fig. 101 must

have a pentagonal hemihedral arrangement of its molecules, since it is evident that the secondary symmetry planes are lacking, while the principal symmetry planes are present. Similarly, Fig. 102 must represent a tetrahedral hemihedral cube, since

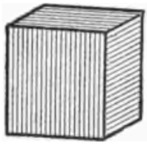


FIG. 101. — Cube striated by a pentagonal dodecahedron.

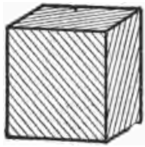


FIG. 102. — Cube striated by a positive tetrahedron.

the secondary symmetry planes are present and the principal symmetry planes are absent.

In order to determine the name of any form striating another form, it is only necessary to ascertain the name of a face which would intersect the striated face in an edge parallel to one of the striations. For instance, it is evident that the pentagonal hemihedral cube shown in Fig. 101 is striated by a face sloping directly toward the observer and parallel to the right and left crystal axis. Such a form is either a dodecahedron or a pentagonal dodecahedron. As a face of the latter form will intersect a cube face in a blunter angle than that formed by the intersection of a dodecahedron and a cube, one is safe in assuming that in the case under consideration the striations are produced by the oscillation of cube and pentagonal dodecahedron faces.

Striations Produced by Oscillatory Twinning: If successive twinning planes in an oscillatory twin

are very close together, a series of ridges and depressions or terraces may be formed on some of the faces of a crystal, as illustrated in Fig. 97. When such ridges or terraces are very narrow, they bear a very close resemblance to striations produced by oscillation of faces. In fact, they cannot always be distinguished from such striations, but they may differ therefrom in that twinning striations may cross each other and may produce a cross-hatched appearance, they cannot be parallel to a symmetry plane (excepting in the case of tetrahedral hemihedral isometric and sphenoidal hemihedral tetragonal crystals), and they may cross a symmetry plane obliquely. More important still is the fact that striations produced by oscillation of faces are confined to the surface of a crystal, while striations produced by oscillatory twinning may be shown equally well upon some cleavage faces.

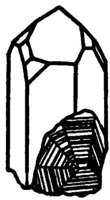


FIG. 103.—Quartz crystal showing striations produced by interference.

Striations Produced by Interference: When two crystals develop in contact with each other the surface between them is sometimes striated in an irregular and peculiar fashion, difficult to describe, but fairly well illustrated by Fig. 103. These striations are usually coarse, and appear to be utterly independent of the symmetry of the crystal.

Cleavage.

Cleavage Defined: Cleavage is the result of a tendency shown by many crystalline substances to

split more or less easily parallel to one or more possible crystal faces. In some cases this tendency is so well developed that the cleavage surfaces are almost as smooth and highly polished as crystal faces, with which cleavage surfaces are sometimes confused.

If cleavage exists parallel to one face of a given crystal form, it is always possible to develop it parallel to every other face of that same form; and, not infrequently, cleavages parallel to the faces of two different forms may be developed on any one crystal.

Cleavage surfaces may be distinguished ordinarily from crystal faces by the fact that they are not usually perfectly flat, but appear to be covered with or made up from very thin sheets or plates, often with curving edges.

Crystal Habit.

The general shape assumed by a crystal is called its habit. Among the commoner terms descriptive of habits are the following:

1. *The name of some crystal form:* For instance, it may be said that a crystal has an octahedral habit when it has the general shape of an octahedron no matter how many other forms are present, or whether the octahedron itself is actually present or absent.

2. *Tabular habit:* This term may be applied to any crystal having the shape of a tablet — an object with two dimensions much greater than the third.

3. *Prismatic habit:* This term may be applied

to any crystal greatly elongated in any one direction no matter whether that direction be parallel to a prism or not. An *acicular* (needle-like) habit is merely an extreme development of a prismatic habit.

Distortion of Crystals.

Most of the statements already made relative to crystals apply only to those which are geometrically perfect, that is, those completely bounded on all sides with faces which in the case of any one form are identically of the same shape and size, and are equally distant from the center of the crystal. Any departure from this condition is known as distortion, and it is unfortunately true that a great majority of crystals are more or less distorted. Two kinds of distortion are recognized, and these are known, respectively, as (1) mechanical distortion, and (2) crystallographic distortion.

Mechanical Distortion: Mechanical distortion is produced by pressure on a completed crystal. This may not only change the molecular arrangement, but it may also tend to flatten a crystal and to bend or warp both it and some or all of the bounding faces. This alters the shape and size of some or all of the faces and their distances from the center of the crystal, and may also warp the faces and *change the angles* which they make with each other.

Although mechanical distortion causes a crystal to depart from all the crystallographic laws already given, this departure is often of so slight a nature as to make it possible to make a fairly accurate guess as to the original appearance of the crystal, and thus to determine the system to which it belongs and the

forms represented upon it. Fortunately this type of distortion is comparatively uncommon.

Crystallographic Distortion: Crystallographic distortion may be conceived to be produced by moving

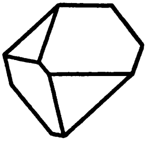


FIG. 104. — Crystallographically distorted octahedron.

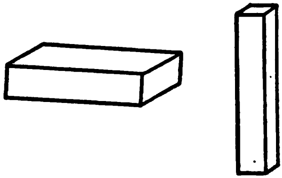


FIG. 105. — Crystallographically distorted cubes.

one or more faces parallel to themselves to any extent either toward or away from the center of the crystal. This may change greatly the shape and size of the faces. In fact, it may result in all the faces of any one form differing from each other in shape and size. Fig. 104 represents such a distorted octahedron, while Fig. 105 represents distorted cubes. In the latter case opposite and parallel faces are of the same shape and size, but differ in these particulars from adjacent faces.

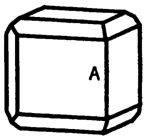


FIG. 106.—Cube modified by a dodecahedron with one face of the latter (which should truncate edge *A*) suppressed.

Not infrequently a face seems to have been moved outward to such an extent as to have been completely crowded off the crystal. This is illustrated in Fig. 106, and faces thus destroyed are said to be *suppressed.*

Although the shape and size of faces and their distances from the center of the crystal are changed

by crystallographic distortion, it is important to remember that the *angles* which such faces make with each other and with the crystal axes *remain unchanged*, and that the angles between edges are likewise unaltered. This follows from the fact that there is no change in the arrangement of the molecules.

From the statements just made it is evident that the definitions of secondary and principal symmetry planes already given cannot be applied to crystallographically distorted crystals. For these, it is necessary to substitute the following definition of a symmetry plane:

Any plane through a crystal is a symmetry plane if there are approximately the same number of faces on opposite sides of this plane, if most of these faces are arranged in pairs on opposite sides of and equally inclined to this plane, and if any two adjacent faces on one side of this plane are usually balanced on the other side by two adjacent faces making identically the same angle with each other. While this rule is not very rigid, and some mistakes may be made in its application, these will be rare exceptions after the student has studied crystals for some time.

It is fortunately true that crystals subject to crystallographic distortion are apt to occur in groups rather than in isolated individuals, and that some members of such groups are apt to be much less distorted than others. In fact, a little search will usually reveal one or more crystals almost geometrically perfect in development, and it is upon these that the attention should be fixed.

In examining crystallographically distorted crys-

tals, it will be found useful to observe the following two rules: (1) All the faces of any one form on a crystal will be of exactly the same color, luster, and smoothness; and, if any one face is striated, all will be striated and in a similar fashion. (2) Where suppression of faces has occurred it is often impossible to decide whether a form is holohedral or its hemihedral or tetartohedral equivalent, as, for instance, a 1st order pyramid or a rhombohedron. In such cases, always assume that the form under consideration is the one which would necessitate the least suppression of faces.

As an illustration of the latter rule, consider a hexagonal crystal showing a 1st order prism capped with six pyramidal faces in the 1st order position, which may represent either a 1st order pyramid or a + and a − rhombohedron. Suppose that a single face in the position of a 2nd order pyramid is found at one of the corners formed by the intersection of two pyramidal and two prismatic faces. Such a crystal is illustrated in Fig. 107. The pyramidal face in the 2nd order position can evidently be interpreted either as a 2nd order pyramid or a 2nd order trigonal pyramid. If we decide the former to be the correct explanation, we must assume the suppression of eleven faces; while if we incline toward the latter possibility, we need assume the suppression of but five faces. According to the rule just laid down, we should

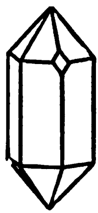


FIG. 107.—Quartz crystal showing suppression of 2nd order trigonal pyramid faces.

make the second assumption, and call the form a 2nd order trigonal pyramid, making the crystal trapezohedral tetartohedral.

In conclusion, it should be remembered that a majority of crystals are not bounded by faces on all sides, but are attached to some foreign substance or to other crystals. This means that a considerable proportion of the surface of most crystals will not bear crystal faces. Such crystals can hardly be called distorted, but the condition mentioned naturally adds to the difficulties involved in their classification.

Vicinal Forms.

The law of rationality of parameters already given states that parameters are always *rational*, fractional or whole, *small* or infinite numbers. While this law holds for all the more prominent faces on crystals that are not mechanically distorted, there sometimes occur on such faces rather inconspicuous elevations, often curved, which accord in form with the symmetry of the face on which they are found, but which are made up of faces that have parameters which are either large, irrational, or both large and irrational. Forms possessing such parameters are termed vicinal. Their cause is not understood. They may be ignored unless unusually prominent, but their form and distribution are sometimes of service in determining the degree of symmetry of crystals.

Etched Figures and Corrosion.

Natural solutions sometimes attack or corrode the plane surfaces bounding crystals. When this hap-

pens the edges may be rounded and the faces curved, and some or all faces may show small triangular, quadrilateral, or polygonal, flat-faced depressions or pits called etched figures. These differ in shape on crystals of different minerals and even on the faces of different forms on an individual crystal. In fact, one form on a crystal may show well-developed etched figures while others are unattacked or merely smoothly corroded. In all cases, however, the shape of the etched figures accords with the symmetry of the face on which they occur, and a study of such figures will sometimes prove helpful in determining the degree of symmetry of crystals showing them.

SYSTEMS, DIVISION OF SYSTEMS, AND FORMS TABULATED

Isometric System

Holohedral Forms

Octahedron PYRITE
Trisoctahedron
Dodecahedron GARNET
Trapezohedron
Hexahedron (cube) GALENA
Hexoctahedron
Tetrahexahedron

Tetrahedral Hemihedral Forms

±Tetrahedron
±Trigonal Tristetrahedron
±Tetragonal Tristetrahedron
±Hextetrahedron

- - - - - - - - - - - - - -

Hexahedron (cube)
Dodecahedron
Tetrahexahedron

Pentagonal Hemihedral Forms

Pentagonal Dodecahedron
Diploid

- - - - - - - - - - - - - -

Octahedron
Dodecahedron
Hexahedron (cube)
Trapezohedron
Trisoctahedron

Gyroidal Hemihedral Forms

Pentagonal Icositetrahedron

- - - - - - - - - - - - - -

Octahedron
Trisoctahedron
Dodecahedron
Trapezohedron
Hexahedron (cube)
Tetrahexahedron

Pentagonal Tetartohedral Forms

Tetartoid

- - - - - - - - - - - - - -

Octahedron
Trisoctahedron
Dodecahedron
Trapezohedron
Hexahedron (cube)
Tetrahexahedron

Hexagonal System

Holohedral Forms

1st Order Pyramid
1st Order Prism
2nd Order Pyramid
2nd Order Prism
Dihexagonal Pyramid
Dihexagonal Prism
Basal-pinacoid

Rhombohedral Hemihedral Forms

±Rhombohedron
Hexagonal Scalenohedron

- - - - - - - - - - - - - -

1st Order Prism
2nd Order Pyramid
2nd Order Prism
Dihexagonal Prism
Basal-pinacoid

Pyramidal Hemihedral Forms

3rd Order Pyramid
3rd Order Prism

- - - - - - - - - - - - - -

1st Order Pyramid
1st Order Prism
2nd Order Pyramid
2nd Order Prism
Basal-pinacoid

Trigonal Hemihedral Forms

±1st Order Trigonal Pyramid
±1st Order Trigonal Prism
Ditrigonal Pyramid
Ditrigonal Prism

- - - - - - - - - - - - - -

2nd Order Pyramid
2nd Order Prism
Basal-pinacoid

Trapezohedral Hemihedral Forms

Hexagonal Trapezohedron

- - - - - - - - - - - - - -

1st Order Pyramid
1st Order Prism
2nd Order Pyramid
2nd Order Prism
Dihexagonal Prism
Basal-pinacoid

Trapezohedral Tetartohedral Forms

±Rhombohedron
±2nd Order Trigonal Pyramid
±2nd Order Trigonal Prism
Trigonal Trapezohedron
Ditrigonal Prism

- - - - - - - - - - - - - -

1st Order Prism
Basal-pinacoid

Rhombohedral Tetartohedral Forms

1st Order Rhombohedron
2nd Order Rhombohedron
3rd Order Rhombohedron
3rd Order Prism

- - - - - - - - - - - - - -

1st Order Prism
2nd Order Prism
Basal-pinacoid

Tetragonal System

Holohedral Forms

1st Order Pyramid
1st Order Prism
2nd Order Pyramid
2nd Order Prism
Ditetragonal Pyramid
Ditetragonal Prism
Basal-pinacoid

Sphenoidal Hemihedral Forms

±Tetragonal Sphenoid
±Tetragonal Scalenohedron

- - - - - - - - - - - - -

1st Order Prism
2nd Order Pyramid
2nd Order Prism
Ditetragonal Prism
Basal-pinacoid

Pyramidal Hemihedral Forms

3rd Order Pyramid
3rd Order Prism

- - - - - - - - - - - - -

1st Order Pyramid
1st Order Prism
2nd Order Pyramid
2nd Order Prism
Basal-pinacoid

Orthorhombic System

Holohedral Forms

Pyramid
Prism
Macro-dome
Brachy-dome
Macro-pinacoid
Brachy-pinacoid
Basal-pinacoid

Sphenoidal Hemihedral Forms

Orthorhombic Sphenoid

- - - - - - - - - - - - -

Prism
Macro-dome
Brachy-dome
Macro-pinacoid
Brachy-pinacoid
Basal-pinacoid

Monoclinic System

Holohedral Forms

±Pyramid
Prism
Clino-dome
Ortho-pinacoid
±Ortho-dome
Basal-pinacoid
Clino-pinacoid

Triclinic System

Holohedral Forms

Same as in the orthorhombic system

INDEX

octahedral
pyramid - rhombohedron
pyramid
prism
unit "
[illegible] - [illegible] - basal - pinacoid

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